CAMPION SIXTH FORM

INDUCTION DAY
FRIDAY 20th JUNE 2025

Campion School Sixth Form

The best in everyone™

A Level Entry Requirements – September 2025

In order to study A Levels, you will need a minimum of 6 GCSEs at grade 5 or above across a range of subjects including Maths and English or 5 GCSEs at grade 6 or above across a range of subjects including Maths and English.

Please note, if you wish to study 4 A Level courses, you need to have achieved Grade 8s and 9s in all GCSEs studied. (Further Mathematics requires a Grade 8).

In addition to the above, you need to meet the subject specific entry requirement to enrol on the course.

Subject	Entry Requirement
Art	4 in English and Maths and 5 in Art and/or DT
Biology	6 in Maths, 5 in English and 7 in Science
Business Studies (A Level)	5 in Maths and English
Chemistry	7 in Maths, 5 in English and 7 in Science
Economics	5 in Maths and 6 in English
English Literature	4 in Maths, 6 in English Literature or 5 in English Language
English Language/Literature	4 in Maths and a combination of a Grade 5 and a Grade in 6 in English
Further Mathematics	8 in Maths and 5 in English
Geography	4 in Maths, 5 in English and 5 in Geography
History	4 in Maths, 5 in English and 5 in History
Mathematics	7 in Maths and 5 in English
Physical Education	4 in Maths, 4 in English and 5 in PE ***Needs to be actively involved in a sporting activity
Physics	7 in Maths, 5 in English and 7 in Science
Politics	4 in Maths, 5 in English and 5 in any Humanities subjects
Psychology	5 in Maths, English and Science
Sociology	4 in Maths, 5 in English and 5 in any Humanities subjects
Criminology (Level 3 Diploma)	4 in Maths and English

Y12 A level Art
Journal Transition Project

PART 1: Visual Journal

Y12 A level Art
Journal Transition Project

What?

Create a visual art journal which reflects aspects of your own life and personality in a small sketchbook or altered book.

How?

Use a combination of drawings, paintings and text/words to illustrate the suggested themes using a range of materials, techniques and processes.

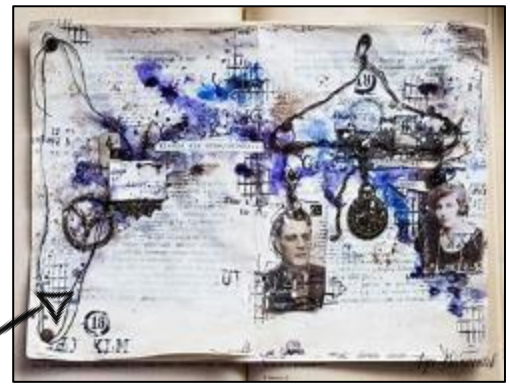
Why?

To develop and refine your observation drawing skills by looking more closely at things around you that you see every day, and to explore skills in creating drawings from imagination.

Sketchbook and Journal ideas

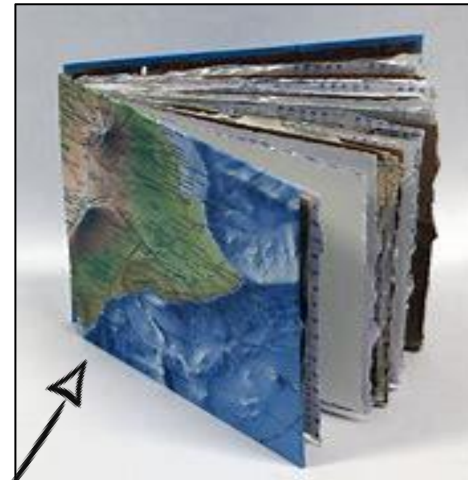
To begin with you will need a sketchbook, try to use one that is no bigger than A4 in size to create your journal as this is a good size. It's also just as easy to make your own - look at some of these examples for ideas. Just use whatever you have available at home. Check out the links and look on YouTube for ideas linked to handmade artist sketchbooks.

You can use an old, damaged or unwanted paper or hardback book and draw on top of the pages and words



Create an accordion or concertina journal by folding and gluing paper into a long strip.

https://www.accessart.org.uk/sketchbooks-an-online-course-for-children-families-parents-teachers/?fbclid=IwAR0eXgOhXUnmJdtsLP-0-SaSrYINphDO0pQP2KIcHuOGrEHLQMQpDJc2IYk



Cardboard cover folded in half with a range of papers inside. Bound together by wrapping some string or an elastic band around the middle.

What will I draw?

Each double page will have a theme (we have given you 30 to start with). You can draw from direct observation, use your own photos or find images on the internet as inspiration. Look at the slides below with artists who use sketchbooks and journals.

You can add more pages and continue it over the Summer too, ready to bring back in September.

You can also collage things into your journal and draw on top of them

Receipts
Tickets
Wrappers
Scrap paper
Envelopes
Postcards
Flyers
Newspapers

Page inspiration and layout ideas

You can use small titles using a relevant font as a way to give context to your page

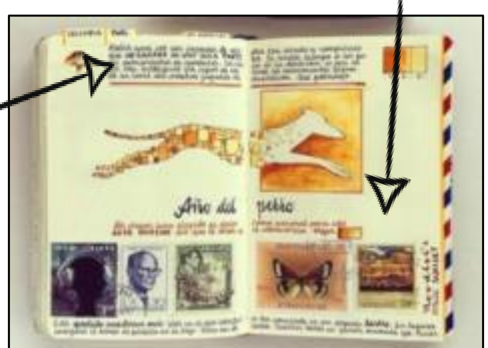
Use annotation to write notes or add further information about the things you have drawn

Use more than one media or technique per double page. Try using different techniques, such as cross hatching, continuous line, detailed tonal studies etc

You can write down the location, date and the time that you do your page entry.

Think about literacy when you annotate your pages, and use these to create a flow and visual story which travels across different pages.

Use a range of different size drawings on the page and add small images and boxes



Think carefully about layout and composition.Some pages can be filled with images and others can have lots of empty space.

Jose Naranja

Jose Naranja turns ordinary notebooks into highly detailed works of art. The artist uses watercolours, stamps, writing, elements of photography and drawings to turn each notebook into a one of a kind masterpiece.

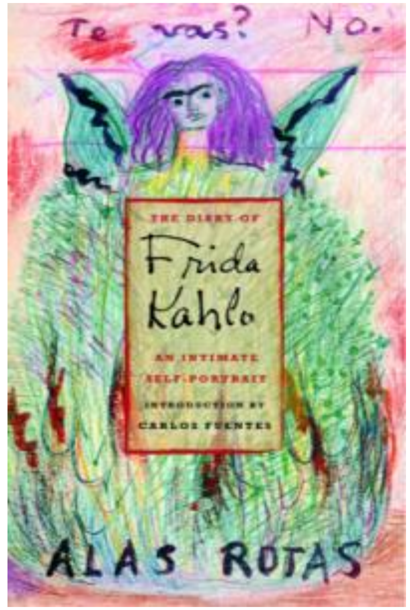
Naranja worked as an aeronautical engineer for many years but eventually decided to quit and devote his life to art and travels. The artist developed his passion for notebook art in 2005 when he discovered Moleskine pocket journals. As hard as it may be to believe, Jose is a self-taught artist, but his skills are remarkable. So far the artist has filled in 12 notebooks and recently has begun binding his own, to make his art even more personal.

http://josenaranja.blogspot.com/

https://www.thisiscolossal.com/2018/04/handmade-sketchbooks-by-jose-naranja/

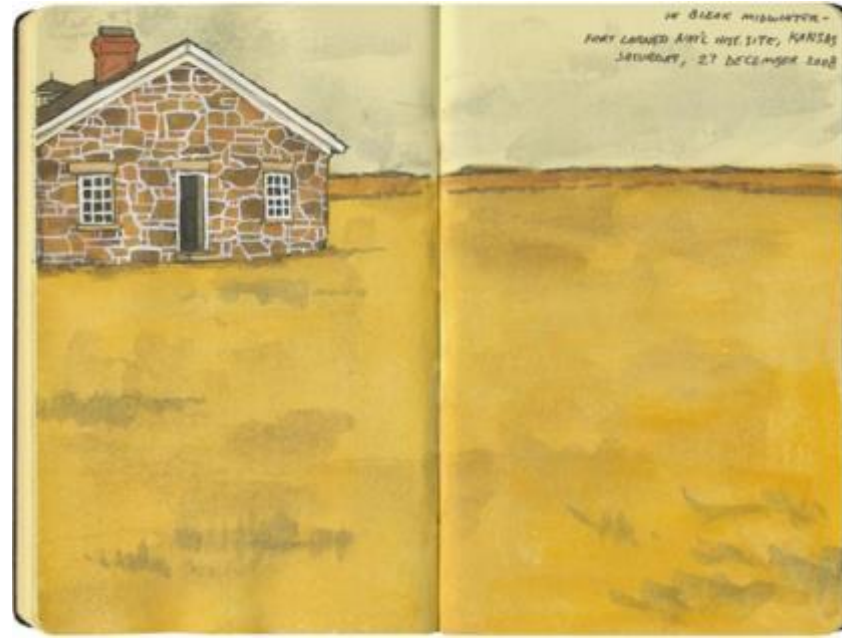
Frida Kahlo

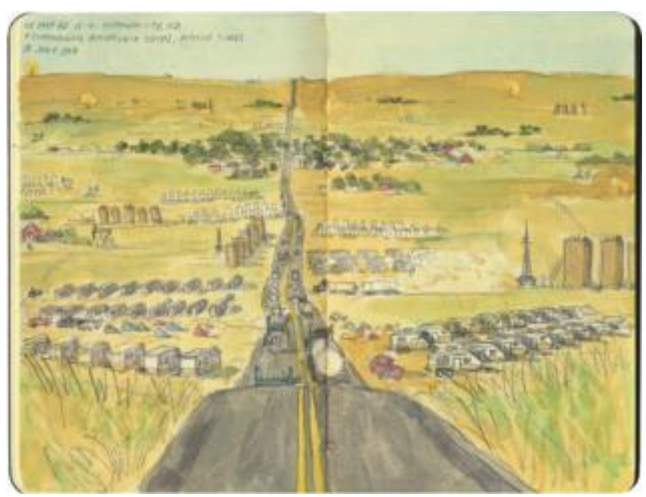
Frida Kahlo's life was expressed through her work. A chronological look at her artwork provides an understanding of the events that changed her life: her passions, motivations, disappointments, and desires. Painting was cathartic for her, however, writing and keeping a diary also helped her to establish a relationship with herself, and to find a way of expressing her afflictions during the final 10 years of her life.



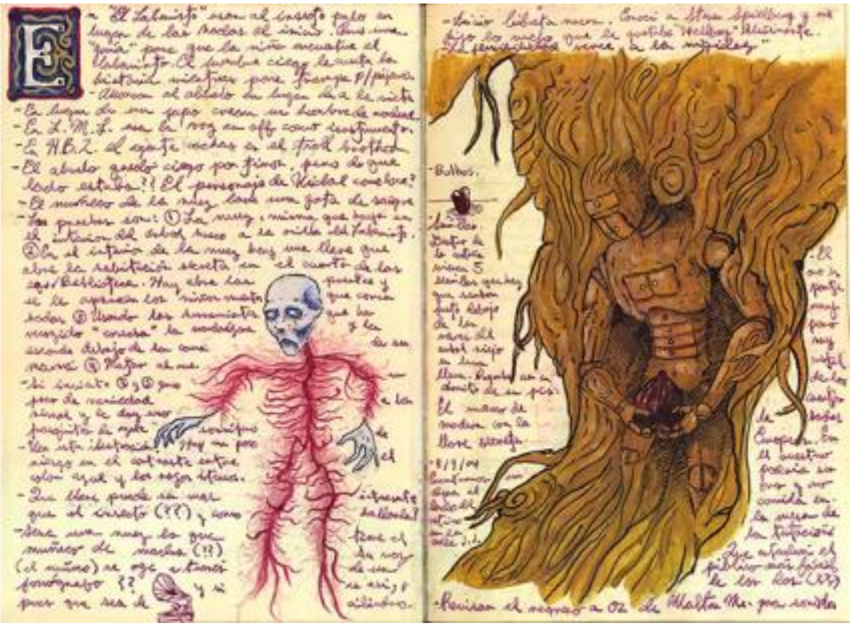
Kahlo found that writing, as well as painting, was useful not just for communicating with her family and friends—and also as a way of connecting with her own feelings, conveying her ideas on her artistic practice, and expressing her worries and pains, both physical and emotional.

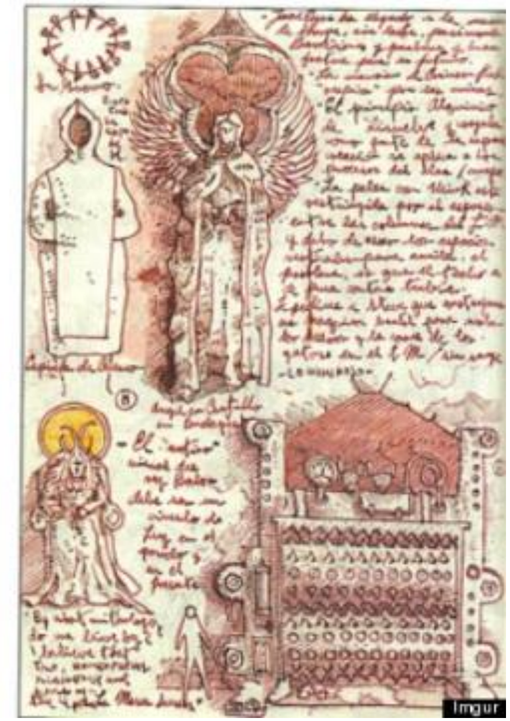
Chandler O'Leary

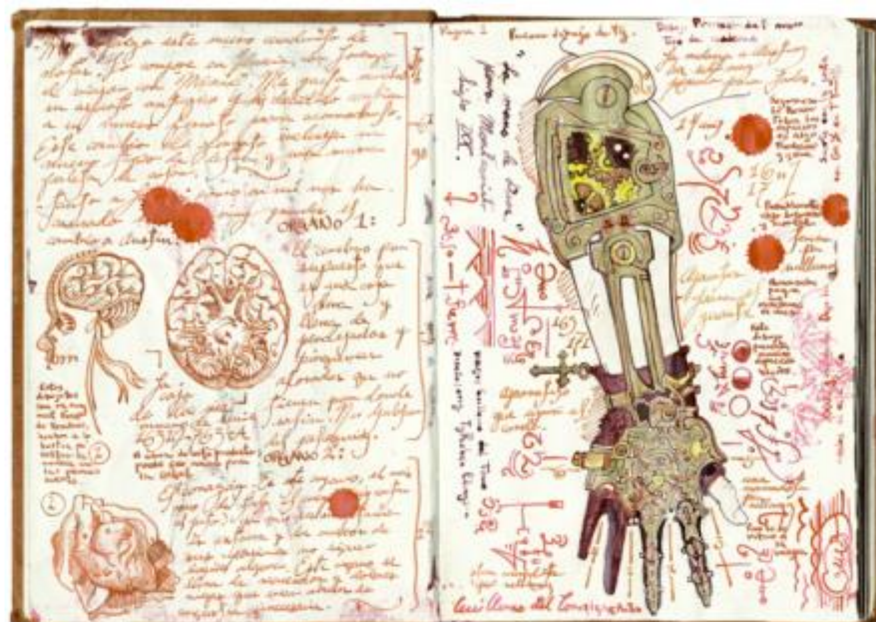




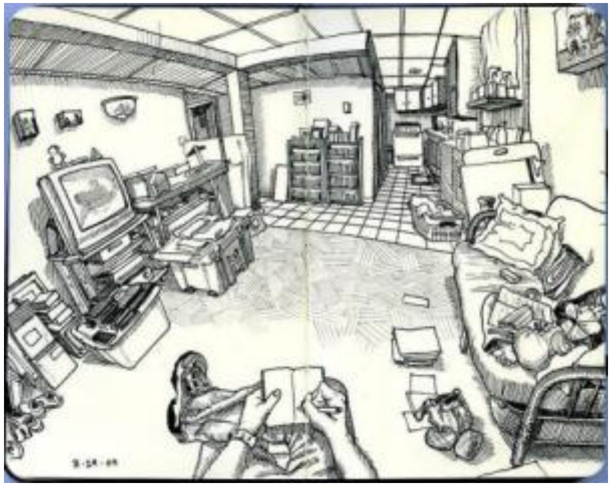
Guillermo Del Toro







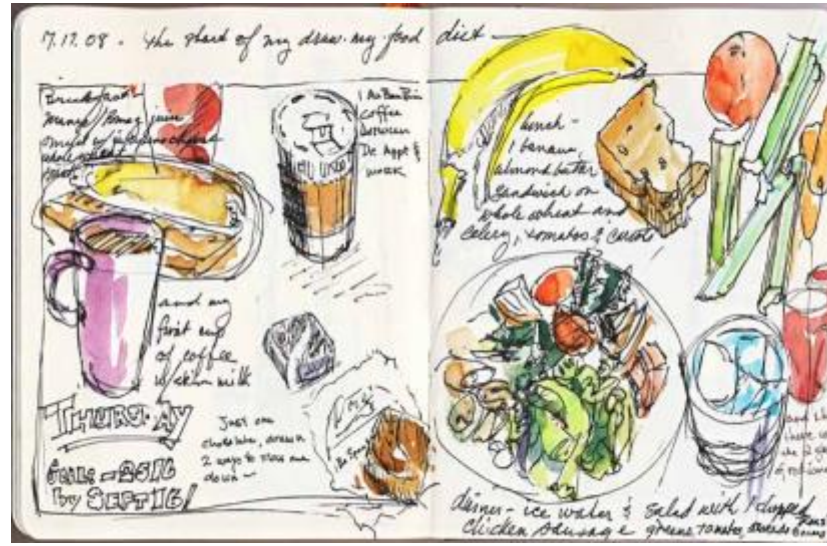
Page 1
Inside

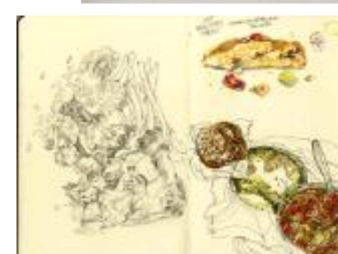


Page 2
Outside

Page 3
Travel and Places

Page 4
Food



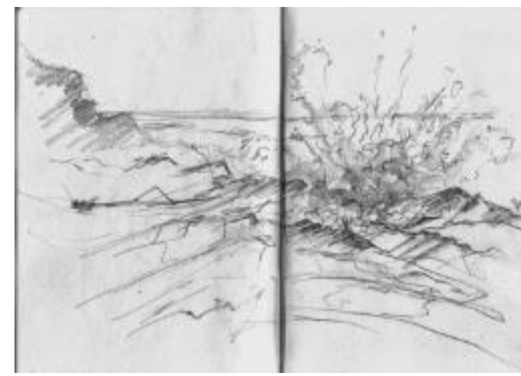


Page 5
Doors

Page 6
Windows

Page 7
Trees

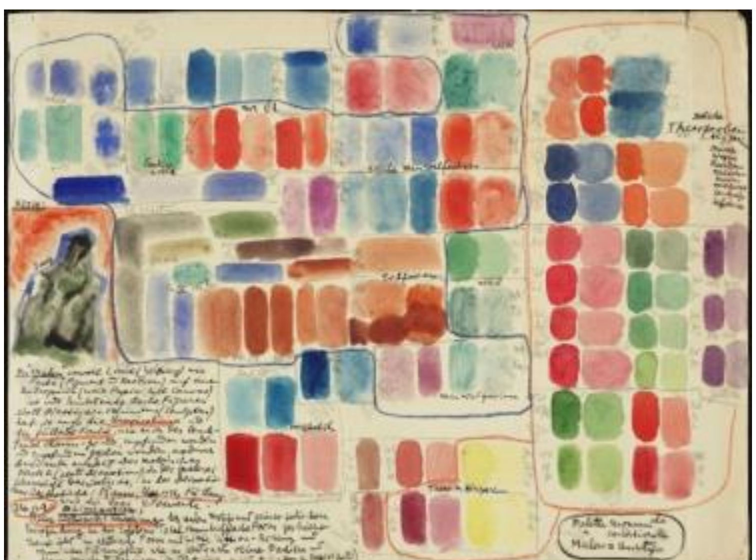
Page 8
Water



Page 9
Possessions

Page 10
Childhood Memories

Page 11
Colour



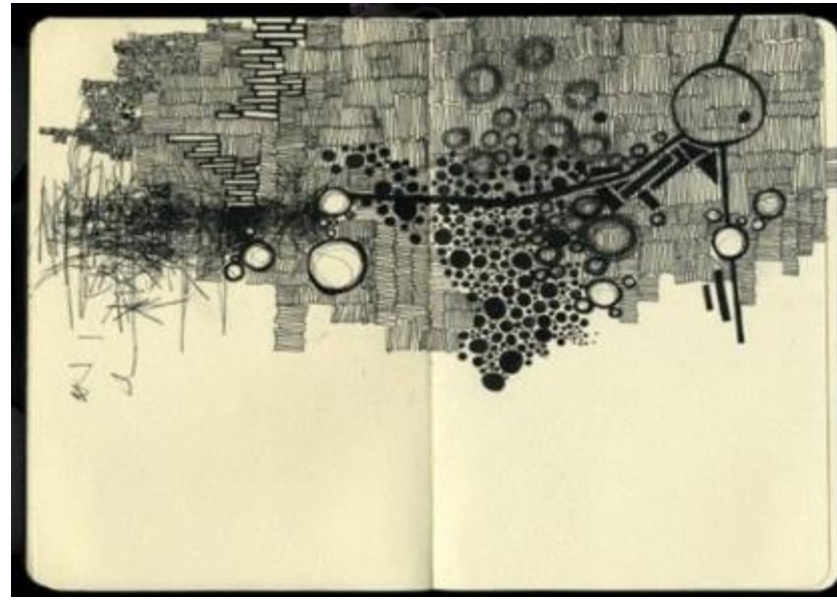
Page 12
Nature

Page 13
Animals

Page 14
Family

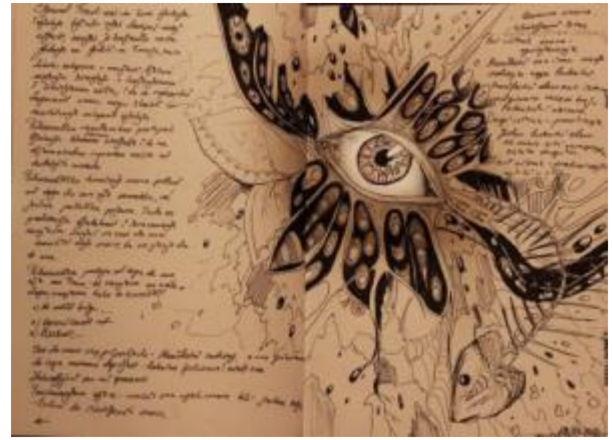
Page 15
The Weather

Page 16
Textures



Page 17
Favourite Film

Page 18
Favourite Book



Page 19
Feelings and Emotions

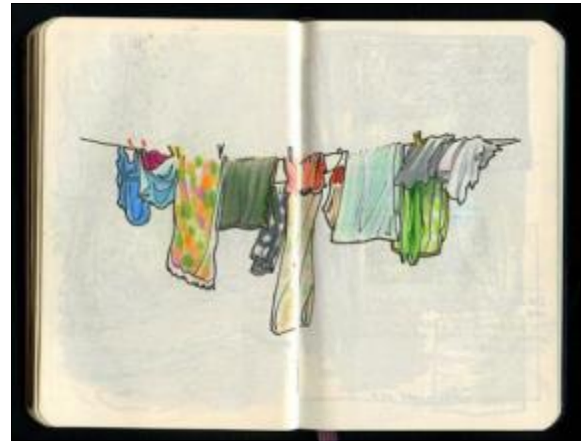
Page 20
Smells

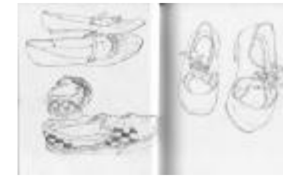
Page 21
Dreams

Page 22
Fears

Page 23
Words

Page 24
Clothes



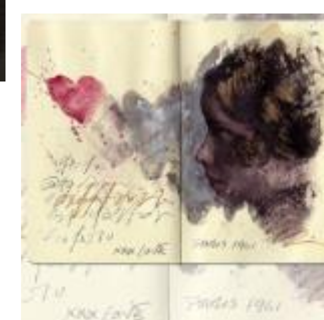


Page 25
Favourite Artist

Page 26
Favourite drawing tools

Page 27
The News

Page 28
Conflict



Page 29
Favourite songs

Page 30
Self Portrait

Transition Pack for A Level Biology

Get ready for A-level!

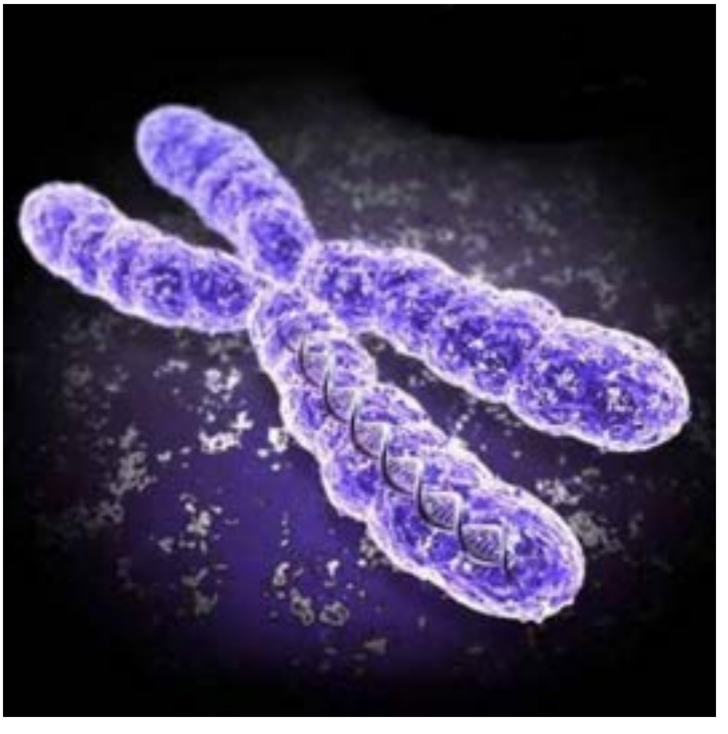
A guide to help you get ready for A-level Biology, including everything from topic guides to days out and online learning courses.

Commissioned by The PiXL Club Ltd. April 2016

Please note: these resources are non-board specific. Please direct your students to the specifics of where this knowledge and skills most apply.

So you are considering A level Biology?

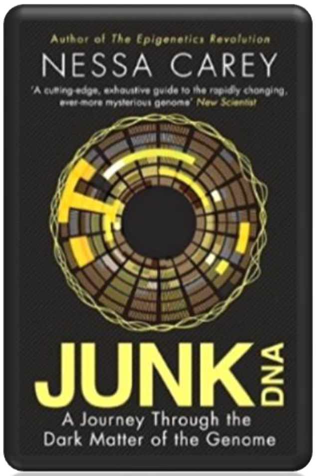
This pack contains a programme of activities and resources to prepare you to start A level in Biology in September. It is aimed to be used after you complete your GCSE throughout the remainder of the Summer term and over the Summer Holidays to ensure you are ready to start your course in September.



https://www.distance-education-academy.com/wp-content/uploads/2013/06/biology-a-level-course.jpg

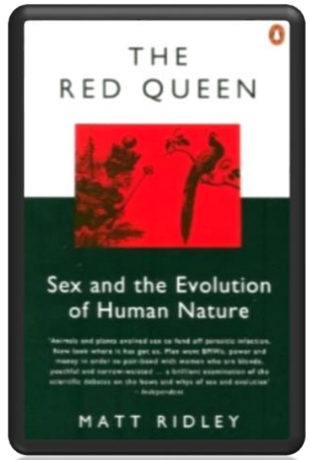
Book Recommendations

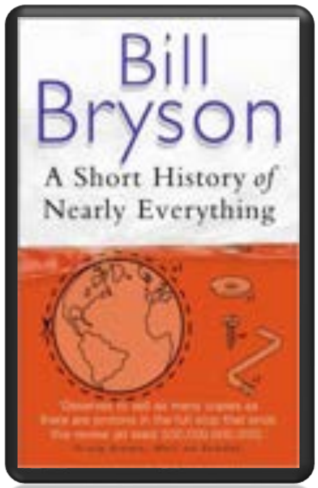
Kick back this summer with a good read. The books below are all popular science books and great for extending your understanding of Biology



Junk DNA
Our DNA is so much more complex than you probably realize, this book will really deepen your understanding of all the work you will do on Genetics. Available at amazon.co.uk

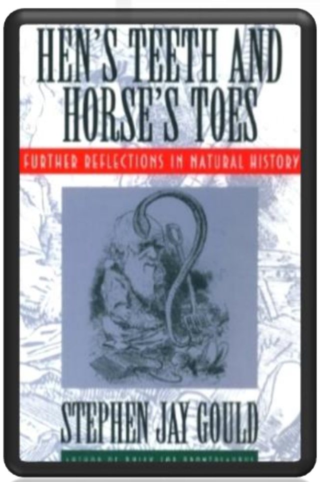
The Red Queen
Its all about sex. Or sexual selection at least. This book will really help your understanding of evolution and particularly the fascinating role of sex in evolution. Available at amazon.co.uk

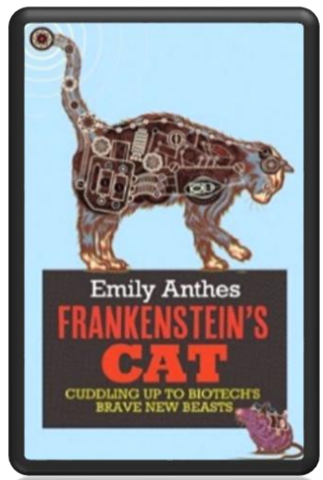




A Short History of Nearly Everything
A whistle-stop tour through many aspects of history from the Big Bang to now. This is a really accessible read that will re-familiarise you with common concepts and introduce you to some of the more colourful characters from the history of science! Available at amazon.co.uk

Studying Geography as well?
Hen's teeth and horses toes
Stephen Jay Gould is a great Evolution writer and this book discusses lots of fascinating stories about Geology and evolution. Available at amazon.co.uk





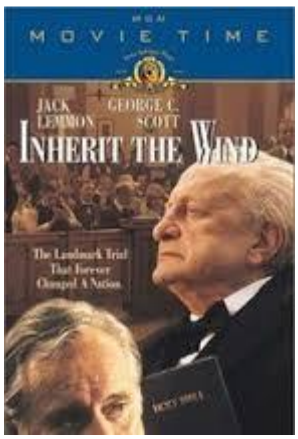
An easy read..
Frankenstein's cat
Discover how glow in the dark fish are made and more great Biotechnology breakthroughs. Available at amazon.co.uk

Movie Recommendations

Everyone loves a good story and everyone loves some great science. Here are some of the picks of the best films based on real life scientists and discoveries. You wont find Jurassic Park on this list, we've looked back over the last 50 years to give you our top 5 films you might not have seen before. Great watching for a rainy day.

Gorillas in the Mist (1988)
An absolute classic that retells the true story of the life and work of Dian Fossey and her work studying and protecting mountain gorillas from poachers and habitat loss. A tear jerker.

Inherit The Wind (1960)
Great if you can find it. Based on a real life trial of a teacher accused of the crime of teaching Darwinian evolution in school in America. Does the debate rumble on today?



Andromeda Strain (1971)
Science fiction by the great thriller writer Michael Cricthon (he of Jurassic Park fame). Humans begin dying when an alien microbe arrives on Earth.

Lorenzo's Oil (1992)
Based on a true story. A young child suffers from an autoimmune disease. The parents research and challenge doctors to develop a new cure for his disease.

Something the Lord Made (2004)
Professor Snape (the late great Alan Rickman) in a very different role. The film tells the story of the scientists at the cutting edge of early heart surgery as well as issues surrounding racism at the time.

There are some great TV series and box sets available too, you might want to check out: Blue Planet, Planet Earth, The Ascent of Man, Catastrophe, Frozen Planet, Life Story, The Hunt and Monsoon.

Movie Recommendations

If you have 30 minutes to spare, here are some great presentations (and free!) from world leading scientists and researchers on a variety of topics. They provide some interesting answers and ask some thought-provoking questions. Use the link or scan the QR code to view:

A New Superweapon in the Fight Against Cancer
Available at :
http://www.ted.com/talks/paula_hammond_a_new_superweapon_in_the_fight_against_cancer?language=en
Cancer is a very clever, adaptable disease. To defeat it, says medical researcher and educator Paula Hammond, we need a new and powerful mode of attack.

Why Bees are Disappearing
Available at :
http://www.ted.com/talks/marla_spivak_why_bees_are_disappearing?language=en
Honeybees have thrived for 50 million years, each colony 40 to 50,000 individuals coordinated in amazing harmony. So why, seven years ago, did colonies start dying en-masse?

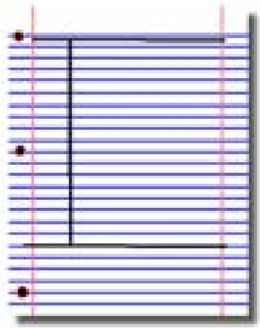
Why Doctors Don't Know About the Drugs They Prescribe
Available at :
http://www.ted.com/talks/ben_goldacre_what_doctors_don_t_know_about_the_drugs_they_prescribe?language=en
When a new drug gets tested, the results of the trials should be published for the rest of the medical world — except much of the time, negative or inconclusive findings go unreported, leaving doctors and researchers in the dark.

Growing New Organs
Available at :
http://www.ted.com/talks/anthony_atala_growing_organs_engineering_tissue?language=en
Anthony Atalla's state-of-the-art lab grows human organs — from muscles to blood vessels to bladders, and more.

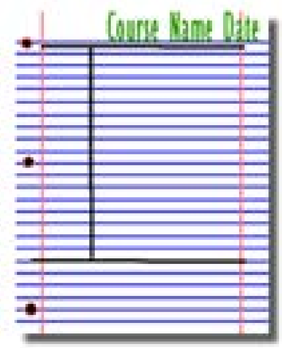
Research activities

Research, reading and note making are essential skills for A level Biology study. For the following task you are going to produce ‘Cornell Notes’ to summarise your reading.

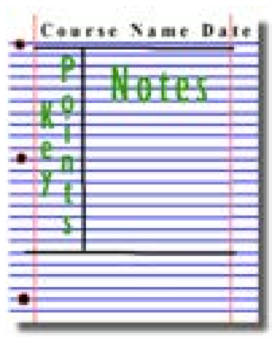
1. Divide your page into three sections like this



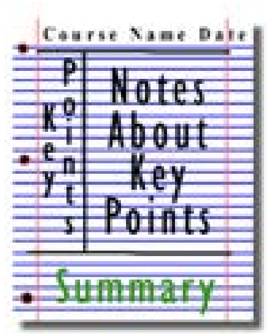
2. Write the name, date and topic at the top of the page



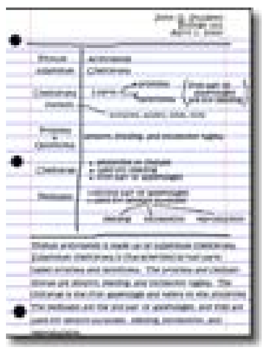
3. Use the large box to make notes. Leave a space between separate idea. Abbreviate where possible.



4. Review and identify the key points in the left hand box



5. Write a summary of the main ideas in the bottom space



Images taken from http://coe.jmu.edu/learningtoolbox/cornellnotes.html

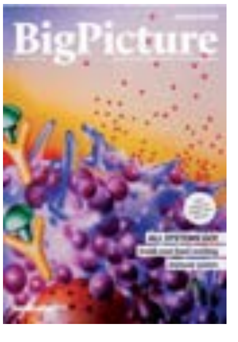
Research activities

The Big Picture is an excellent publication from the Wellcome Trust. Along with the magazine, the company produces posters, videos and other resources aimed at students studying for GCSEs and A level.
For each of the following topics, you are going to use the resources to produce one page of Cornell style notes.
Use the links of scan the QR code to take you to the resources.

BigPicture

Topic 1: The Cell
Available at: http://bigpictureeducation.com/cell
The cell is the building block of life. Each of us starts from a single cell, a zygote, and grows into a complex organism made of trillions of cells. In this issue, we explore what we know – and what we don't yet know – about the cells that are the basis of us all and how they reproduce, grow, move, communicate and die.

Topic 2: The Immune System
Available at:
http://bigpictureeducation.com/immune
The immune system is what keeps us healthy in spite of the many organisms and substances that can do us harm. In this issue, explore how our bodies are designed to prevent potentially harmful objects from getting inside, and what happens when bacteria, viruses, fungi or other foreign organisms or substances breach these barriers.



Topic 3: Exercise, Energy and Movement
Available at:
http://bigpictureeducation.com/exercise-energy-and-movement
All living things move. Whether it's a plant growing towards the sun, bacteria swimming away from a toxin or you walking home, anything alive must move to survive. For humans though, movement is more than just survival – we move for fun, to compete and to be healthy. In this issue we look at the biological systems that keep us moving and consider some of the psychological, social and ethical aspects of exercise and sport.

Topic 4: Populations
Available at:
http://bigpictureeducation.com/populations
What's the first thing that pops into your mind when you read the word population? Most likely it's the ever-increasing human population on earth. You're a member of that population, which is the term for all the members of a single species living together in the same location. The term population isn't just used to describe humans; it includes other animals, plants and microbes too. In this issue, we learn more about how populations grow, change and move, and why understanding them is so important.

Topic 4: Populations
Available at: http://bigpictureeducation.com/health-and-climate-change
The Earth's climate is changing. In fact, it has always been changing. What is different now is the speed of change and the main cause of change – human activities. This issue asks: What are the biggest threats to human health? Who will suffer as the climate changes? What can be done to minimise harm? And how do we cope with uncertainty?

Pre-Knowledge Topics

A level Biology will use your knowledge from GCSE and build on this to help you understand new and more demanding ideas. Complete the following tasks to make sure your knowledge is up to date and you are ready to start studying:

DNA and the Genetic Code

In living organisms nucleic acids (DNA and RNA have important roles and functions related to their properties. The sequence of bases in the DNA molecule determines the structure of proteins, including enzymes.
The double helix and its four bases store the information that is passed from generation to generation. The sequence of the base pairs adenine, thymine, cytosine and guanine tell ribosomes in the cytoplasm how to construct amino acids into polypeptides and produce every characteristic we see. DNA can mutate leading to diseases including cancer and sometimes anomalies in the genetic code are passed from parents to babies in disease such as cystic fibrosis, or can be developed in unborn foetuses such as Downs Syndrome.
Read the information on these websites (you could make more Cornell notes if you wish):
http://www.bbc.co.uk/education/guides/z36mmp3/revision
http://www.s-cool.co.uk/a-level/biology/dna-and-genetic-code

And take a look at these videos:
http://ed.ted.com/lessons/the-twisting-tale-of-dna-judith-hauck
http://ed.ted.com/lessons/where-do-genes-come-from-carl-zimmer

Task:
Produce a wall display to put up in your classroom in September. You might make a poster or do this using PowerPoint or similar Your display should use images, keywords and simple explanations to:
Define gene, chromosome, DNA and base pair
Describe the structure and function of DNA and RNA
Explain how DNA is copied in the body
Outline some of the problems that occur with DNA replication and what the consequences of this might be.

Evolution

Transfer of genetic information from one generation to the next can ensure continuity of species or lead to variation within a species and possible formation of new species. Reproductive isolation can lead to accumulation of different genetic information in populations potentially leading to formation of new species (speciation). Sequencing projects have read the genomes of organisms ranging from microbes and plants to humans. This allows the sequences of the proteins that derive from the genetic code to be predicted. Gene technologies allow study and alteration of gene function in order to better understand organism function and to design new industrial and medical processes.
Read the information on these websites (you could make more Cornell notes if you wish):
http://www.bbc.co.uk/education/guides/z237hyc/revision/4
http://www.s-cool.co.uk/a-level/biology/evolution

And take a look at these videos:
http://ed.ted.com/lessons/how-to-sequence-the-human-genome-mark-j-kiel
http://ed.ted.com/lessons/the-race-to-sequence-the-human-genome-tien-nguyen

Task:
Produce a one page revision guide for an AS Biology student that recaps the key words and concepts in this topic. Your revision guide should:
Describe speciation
Explain what a genome is
Give examples of how this information has already been used to develop new treatments and technologies.

Biodiversity

The variety of life, both past and present, is extensive, but the biochemical basis of life is similar for all living things. Biodiversity refers to the variety and complexity of life and may be considered at different levels. Biodiversity can be measured, for example within a habitat or at the genetic level. Classification is a means of organising the variety of life based on relationships between organisms and is built around the concept of species. Originally classification systems were based on observable features but more recent approaches draw on a wider range of evidence to clarify relationships between organisms. Adaptations of organisms to their environments can be behavioural, physiological and anatomical. Adaptation and selection are major factors in evolution and make a significant contribution to the diversity of living organisms.

Read the information on these websites (you could make more Cornell notes if you wish):
http://www.s-cool.co.uk/a-level/biology/ecological-concepts
http://www.s-cool.co.uk/a-level/biology/classification

And take a look at these videos:
http://ed.ted.com/lessons/why-is-biodiversity-so-important-kim-preshoff
http://ed.ted.com/lessons/can-wildlife-adapt-to-climate-change-erin-eastwood

Task:
Write a persuasive letter to an MP, organisation or pressure group promoting conservation to maintain biodiversity.
Your letter should:
Define what is meant by species and classification
Describe how species are classified
Explain one way scientists can collect data about a habitat, giving an example
Explain adaptation and how habitat change may pose a threat to niche species

Exchange and Transport

Organisms need to exchange substances selectively with their environment and this takes place at exchange surfaces. Factors such as size or metabolic rate affect the requirements of organisms and this gives rise to adaptations such as specialised exchange surfaces and mass transport systems. Substances are exchanged by passive or active transport across exchange surfaces. The structure of the plasma membrane enables control of the passage of substances into and out of cells

Read the information on these websites (you could make more Cornell notes if you wish):
http://www.s-cool.co.uk/a-level/biology/gas-exchange
http://www.s-cool.co.uk/a-level/biology/nutrition-and-digestion/revise-it/human-digestive-system

And take a look at these videos:
http://ed.ted.com/lessons/insights-into-cell-membranes-via-dish-detergent-ethan-perlstein
http://ed.ted.com/lessons/what-do-the-lungs-do-emma-bryce

Task:
Create a poster or display to go in your classroom in September. Your poster should either: compare exchange surfaces in mammals and fish or compare exchange surfaces in the lungs and the intestines. You could use a Venn diagram to do this. Your poster should:
Describe diffusion, osmosis and active transport
Explain why oxygen and glucose need to be absorbed and waste products removed
Compare and contrast your chosen focus.

Cells

The cell is a unifying concept in biology, you will come across it many times during your two years of A level study. Prokaryotic and eukaryotic cells can be distinguished on the basis of their structure and ultrastructure. In complex multicellular organisms cells are organised into tissues, tissues into organs and organs into systems. During the cell cycle genetic information is copied and passed to daughter cells. Daughter cells formed during mitosis have identical copies of genes while cells formed during meiosis are not genetically identical

Read the information on these websites (you could make more Cornell notes if you wish):
http://www.s-cool.co.uk/a-level/biology/cells-and-organelles
http://www.bbc.co.uk/education/guides/zvjycdm/revision

And take a look at these videos:
https://www.youtube.com/watch?v=gcTuQpuJyD8
https://www.youtube.com/watch?v=L0k-enzoeOM
https://www.youtube.com/watch?v=qCLmR9-YY7o

Task:

Produce a one page revision guide to share with your class in September summarising one of the following topics: Cells and Cell Ultrastructure, Prokaryotes and Eukaryotes, or Mitosis and Meiosis.

Whichever topic you choose, your revision guide should include:
Key words and definitions
Clearly labelled diagrams
Short explanations of key ideas or processes.

Biological Molecules

Biological molecules are often polymers and are based on a small number of chemical elements. In living organisms carbohydrates, proteins, lipids, inorganic ions and water all have important roles and functions related to their properties. DNA determines the structure of proteins, including enzymes. Enzymes catalyse the reactions that determine structures and functions from cellular to whole-organism level. Enzymes are proteins with a mechanism of action and other properties determined by their tertiary structure. ATP provides the immediate source of energy for biological processes.

Read the information on these websites (you could make more Cornell notes if you wish):
http://www.s-cool.co.uk/a-level/biology/biological-molecules-and-enzymes
http://www.bbc.co.uk/education/guides/zb739j6/revision

And take a look at these videos:
https://www.youtube.com/watch?v=H8WJ2KENlK0
http://ed.ted.com/lessons/activation-energy-kickstarting-chemical-reactions-vance-kite

Task:

Krabbe disease occurs when a person doesn't have a certain enzyme in their body. The disease effects the nervous system. Write a letter to a GP or a sufferer to explain what an enzyme is.

Your poster should:
Describe the structure of an enzyme
Explain what enzymes do inside the body

Ecosystems

Ecosystems range in size from the very large to the very small. Biomass transfers through ecosystems and the efficiency of transfer through different trophic levels can be measured. Microorganisms play a key role in recycling chemical elements. Ecosystems are dynamic systems, usually moving from colonisation to climax communities in a process known as succession. The dynamic equilibrium of populations is affected by a range of factors. Humans are part of the ecological balance and their activities affect it both directly and indirectly. Effective management of the conflict between human needs and conservation help to maintain sustainability of resources.

Read the information on these websites (you could make more Cornell notes if you wish):
http://www.bbc.co.uk/education/guides/z7vqtfr/revision
http://www.s-cool.co.uk/a-level/biology/ecological-concepts

And take a look at these videos:
https://www.youtube.com/watch?v=jZKIHe2LDP8
https://www.youtube.com/watch?v=E8dkWQVFAoA

Task:
Produce a newspaper or magazine article about one ecosystem (e.g. the arctic, the Sahara, the rainforest, or something closer to home like your local woodland, nature reserve or shore line).
Your article should include:
Key words and definitions
Pictures or diagrams of your chosen ecosystem.
A description of the changes that have occurred in this ecosystem
An explanation of the threats and future changes that may further alter this ecosystem.

Control Systems

Homeostasis is the maintenance of a constant internal environment. Negative feedback helps maintain an optimal internal state in the context of a dynamic equilibrium. Positive feedback also occurs. Stimuli, both internal and external, are detected leading to responses. The genome is regulated by a number of factors. Coordination may be chemical or electrical in nature

Read the information on these websites (you could make more Cornell notes if you wish):
http://www.s-cool.co.uk/a-level/biology/homeostasis
http://www.bbc.co.uk/education/topics/z8kxpv4

And take a look at these videos:
https://www.youtube.com/watch?v=x4PPZCLnVkA
https://www.youtube.com/watch?v=x4PPZCLnVkA

Task:
Produce a poster to display in your classroom in September summarising one of the following topics: Temperature Control, Water and the Kidneys, Glucose, or The Liver.
Whichever topic you choose, your poster or display should include:
Key words and definitions
Clearly labelled diagrams
Short explanations of key ideas or processes.

Energy for Biological Processes

In cellular respiration, glycolysis takes place in the cytoplasm and the remaining steps in the mitochondria. ATP synthesis is associated with the electron transfer chain in the membranes of mitochondria and chloroplasts in photosynthesis energy is transferred to ATP in the light- dependent stage and the ATP is utilised during synthesis in the light-independent stage.

Read the information on these websites (you could make more Cornell notes if you wish):
http://www.bbc.co.uk/education/guides/zcxrd2p/revision
http://www.s-cool.co.uk/a-level/biology/respiration

And take a look at these videos:
https://www.youtube.com/watch?v=00jbG_cfGuQ
https://www.youtube.com/watch?v=2f7YwCtHcgk

Task:

Produce an A3 annotated information poster that illustrates the process of cellular respiration and summarises the key points.

Your poster should include:
Both text and images
Be visually stimulating
Key words and definitions
Clearly labelled diagrams
Short explanations of key ideas or processes.

Scientific and Investigative Skills

As part of your A level you will complete a practical assessment. This will require you to carry out a series of practical activities as well as planning how to do them, analysing the results and evaluating the methods. This will require you to: use appropriate apparatus to record a range of quantitative measurements (to include mass, time, volume, temperature, length and pH), use appropriate instrumentation to record quantitative measurements, such as a colorimeter or photometer, use laboratory glassware apparatus for a variety of experimental techniques to include serial dilutions, use of light microscope at high power and low power, including use of a graticule, produce scientific drawing from observation with annotations, use qualitative reagents to identify biological molecules, separate biological compounds using thin layer/paper chromatography or electrophoresis, safely and ethically use organisms, use microbiological aseptic techniques, including the use of agar plates and broth, safely use instruments for dissection of an animal organ, or plant organ, use sampling techniques in fieldwork.

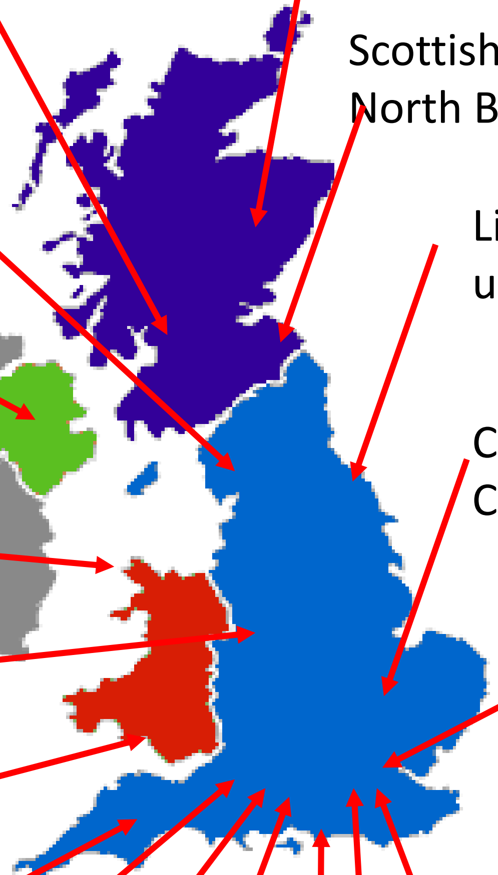
Task:

Produce a glossary for the following key words:

accuracy, anomaly, calibration, causal link, chance, confounding variable, control experiment, control group, control variable, correlation, dependent variable, errors, evidence, fair test, hypothesis, independent, null hypothesis, precision, probability, protocol, random distribution, random error, raw data, reliability, systematic error, true value, validity, zero error,

Ideas for Day Trips

If you are on holiday in the UK, or on a staycation at home, why not plan a day trip to one of these :



Ideas for Day Trips

If you are on holiday in the UK, or on a staycation at home, why not plan a day trip to one of these :

Remember there are also lots of zoos, wildlife and safari parks across the country, here are some you may not have heard of or considered:
Colchester Zoo, Cotswold Wildlife Park, Banham Zoo (Norfolk), Tropical Birdland (Leicestershire), Yorkshire Wildlife Park, Peak Wildlife Park, International Centre for Birds of Prey (York), Blackpool Zoo, Beale Park (Reading)

There are also hundreds of nature reserves (some of which are free) located all over the country including:
RSPB sites at Lochwinnoch, Saltholme, Fairburn Ings, Old Moor, Conwy, Minsmere, Rainham Marshes, Pulborough Brooks, Radipole Lake, Newport Wetlands.
Wildlife Trust Reserves and others at Rutland Water, Pensthorpe, Insh Marshes, Attenborough Centre, Inversnaid, Skomer, Loch Garten, Donna Nook, Chapmans Well, Woodwalton Fen, London Wetland Centre, Martin Down and Woolston Eyes Reserve.

Many organisations also have opportunities for people to volunteer over the summer months, this might include working in a shop/café/visitor centre, helping with site maintenance or taking part in biological surveys. Not only is this great experience, it looks great on a job or UCAS application.

For opportunities keep an eye out in your local press, on social media, or look at the websites of organisations like the RSPB, Wildlife Trust, National Trust or Wildlife & Wetland Trust.

There are also probably lots of smaller organisations near you who would also appreciate any support you can give!

Science on Social Media

Science communication is essential in the modern world and all the big scientific companies, researchers and institutions have their own social media accounts. Here are some of our top tips to keep up to date with developing news or interesting stories:

Follow on Twitter:
Commander Chris Hadfield – former resident aboard the International Space Station @cmdrhadfield

Tiktaalik roseae – a 375 million year old fossil fish with its own Twitter account!
@tiktaalikroseae

NASA's Voyager 2 – a satellite launched nearly 40 years ago that is now travelling beyond our Solar System
@NSFVoyager2

Neil dGrasse Tyson – Director of the Hayden Planetarium in New York
@neiltyson

Sci Curious – feed from writer and Bethany Brookshire tweeting about good, bad and weird neuroscience
@scicurious

The SETI Institute – The Search for Extra Terrestrial Intelligence, be the first to know what they find!
@setiinstitute

Carl Zimmer – Science writer Carl blogs about the life sciences
@carlzimmer

Phil Plait – tweets about astronomy and bad science
@badastronomer

Virginia Hughes – science journalist and blogger for National Geographic, keep up to date with neuroscience, genetics and behaviour
@virginiahughes

Maryn McKenna – science journalist who writes about antibiotic resistance
@marynmck

Find on Facebook:
Nature - the profile page for nature.com for news, features, research and events from Nature Publishing Group

Marin Conservation Institute – publishes the latest science to identify important marine ecosystems around the world.

National Geographic - since 1888, National Geographic has travelled the Earth, sharing its amazing stories in pictures and words.

Science News Magazine - Science covers important and emerging research in all fields of science.

BBC Science News - The latest BBC Science and Environment News: breaking news, analysis and debate on science and nature around the world.

Science websites

These websites all offer an amazing collection of resources that you should use again and again through out your course.

Probably the best website on Biology….
Learn Genetics from Utah University has so much that is pitched at an appropriate level for you and has lots of interactive resources to explore, everything from why some people can taste bitter berries to how we clone mice or make glow in the dark jelly fish.
http://learn.genetics.utah.edu/

In the summer you will most likely start to learn about Biodiversity and Evolution. Many Zoos have great websites, especially London Zoo. Read about some of the case studies on conservation, such as the Giant Pangolin, the only mammal with scales.
https://www.zsl.org/conservation

At GCSE you learnt how genetic diseases are inherited. In this virtual fly lab you get to breed fruit flies to investigate how different features are passed on.
http://sciencecourseware.org/vcise/drosophila/

DNA from the beginning is full of interactive animations that tell the story of DNA from its discovery through to advanced year 13 concepts.
One to book mark!
http://www.dnaftb.org/

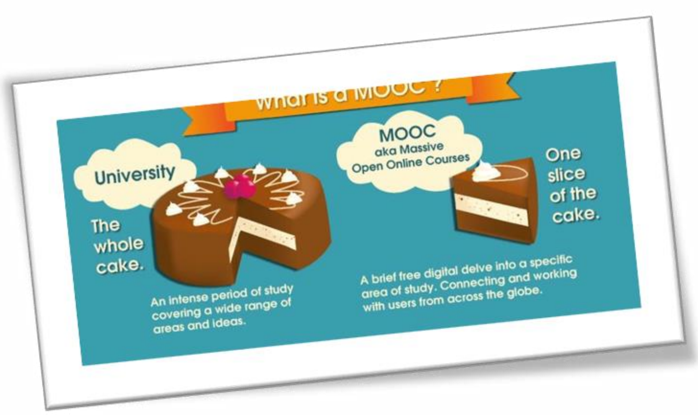
Ok, so not a website, but a video you definitely want to watch. One of the first topics you will learn about is the amazing structure of the cell. This BBC film shows the fascinating workings of a cell… a touch more detailed than the "fried egg" model you might have seen.
http://www.dailymotion.com/video/xzh0kb_the-hidden-life-of-the-cell_shortfilms
If this link expires – google "BBC hidden life of the cell"

Science: Things to do!

Day 4 of the holidays and boredom has set in?
There are loads of citizen science projects you can take part in either from the comfort of your bedroom, out and about, or when on holiday. Wikipedia does a comprehensive list of all the current projects taking place. Google 'citizen science project'

Want to stand above the rest when it comes to UCAS? Now is the time to act.
MOOCs are online courses run by nearly all Universities. They are short FREE courses that you take part in. They are usually quite specialist, but aimed at the public, not the genius!
There are lots of websites that help you find a course, such as edX and Future learn.
You can take part in any course, but there are usually start and finish dates.
They mostly involve taking part in web chats, watching videos and interactives.

Completing a MOOC will look great on your Personal statement and they are dead easy to take part in!



A Level Biology Transition Baseline Assessment

The following 40 minute test is designed to test your recall, analysis and evaluative skills and knowledge.
Remember to use your exam technique: look at the command words and the number of marks each question is worth.
A suggested mark scheme is provided for you to check your answers.

1. a) What are the four base pairs found in DNA?

....
(2)

b) What does DNA code for?

....
(1)

c) Which organelle in a cell carries out this function?

....
(1)

2. a) What theory did Charles Darwin propose?

....
(1)

b) Why did many people not believe Darwin at the time?

....
(1)

c) Describe how fossils are formed.

....

....

....
(3)

d) The fossil record shows us that there have been some species that have formed and some that have become extinct.

i) What is meant by the term 'species'?

....
(2)

ii) Describe how a new species may arise:

....

....

....(3)

3. Ecologists regularly study habitats to measure the species present and the effect of any changes.
One team of ecologists investigated the habitat shown in the picture below:

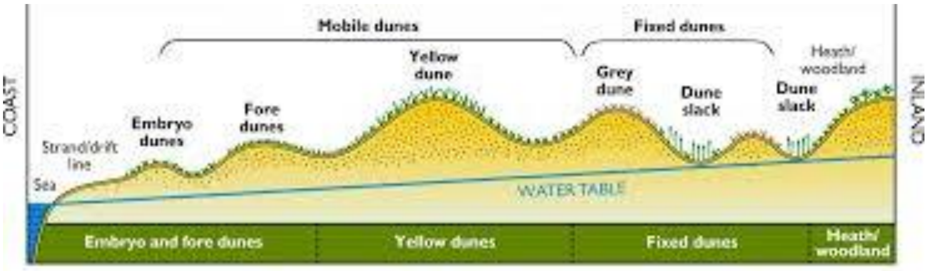


Image taken from http://www.macaulay.ac.uk/soilquality/Dune%20Succession.pdf

a) Define the following keywords:
i) Population

……

ii) Community

……
(2)

b) Give an example of one biotic factor and one abiotic factor that would be present in this habitat

Biotic: ……

Abiotic: ………
(2)

c) Describe how the ecologists would go about measuring the species present between the coast and the inland.

……

……

……

……

……

……
(6)

4. Every living organism is made of cells.

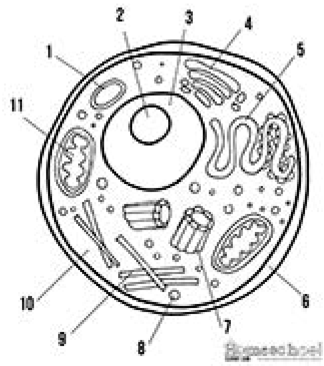


Image taken from http://prestigebux.com/worksheet/label-an-animal-cell-worksheet

a) Label the following parts of the animal cell:

2 ………

5 ………

8 ………

(3)

b) Describe how is the structure of the cell membrane related to its function?

……

……

……

(3)

5. A medical research team investigated how quickly the body deals with glucose after a meal. They studied the blood glucose concentration of people who exercised versus those who did not.
Here are their results:

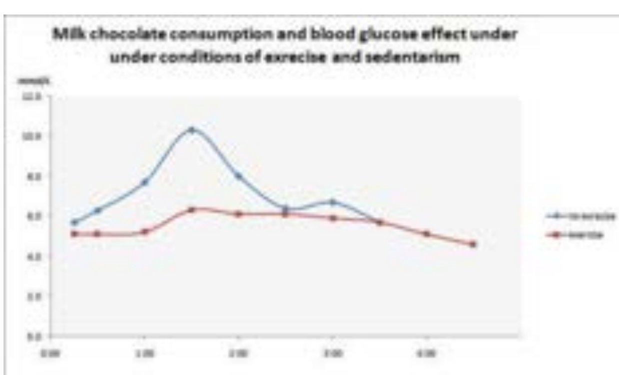


Image taken from https://memoirsofanamnesic.wordpress.com/category/blood-glucose/

a) What organ in the body regulates blood glucose concentration?

……

(1)

b) Explain how the stages that would bring about a return to normal blood glucose concentrations.

……

……

……

……

(4)

c) Name one variable the researchers will have controlled.

……

(1)

d) The researchers made the following conclusion:

"Blood glucose returns to normal values for all people after 4 hours"

To what extent do you agree with this conclusion.

……

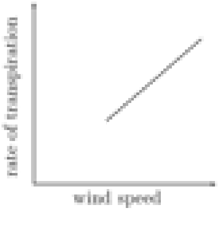
……

……

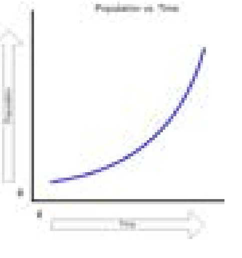
……

(3)

6. Scientists need to be able to interpret data in graphs to decide if there are trends in the results. For each graph bellow, describe the trend.



……………………………………………………………… ………………………………………………………………



……………………………………………………………… ………………………………………………………………(4)

Images taken from: http://www.everythingmaths.co.za/science/lifesciences/grade-10/05-support-and-transport-systems-in-plants/images/56aff2f9b6c5b041688f745ca928990c.png
http://www.bbc.co.uk/staticarchive/afa3f2b16b4d58d077943c96929c9a4020fea83a.gif
http://www.rpi.edu/dept/chem-eng/Biotech-Environ/Projects00/temph/enzyme.html
http://www.myearthwatchexperience.com/Essential%20Ecology.htm

Suggested Mark Scheme:

Question			Answer	Marks
1	a		Adenine-Thymine Cytosine-Guanine	1 1
	b		Protein/enzymes	1
	c		Ribosomes	1
2	a		Evolution (by natural selection)	1
	b		Not enough evidence	1
	c		(Plant/animal dies) and is quickly buried in sediment Not all conditions for decay are present Hard parts of the body are replaced by minerals	1 1 1
	d	i	Organisms that can reproduce to produce viable offspring/offspring that can also reproduce (fertile)	1
		ii	3 from Geographical isolation/named example Mutation of genes Natural Selection/selective advantage Species can no longer interbreed (not produce fertile offspring)	 1 1 1 1
3	a	i	A group of organisms, all of the same species, and all of whom live together in a particular habitat.	1
		ii	The total of all populations living together in a particular habitat.	1
	b		Biotic – one from: Predators, prey, plant, microbes Abiotic – one from: Availability of water, temperature, mineral concentration, reference to climate/weather	 1 1
	c		Measure out a transect Using a tape measure Use a quadrat At regular (named) intervals Identify species present Using a key/guide	1 1 1 1 1 1
4	A		2 Nucleolus 5 Smooth Endoplasmic Reticulum 8 Golgi body	1 1 1

Question			Answer	Marks
4	b		Any 3 from the following structure **and** function must be given. Lipid bilayer - has a hydrophobic inside and hydrophilic outside, allowing for selective permeability Proteins - allow for specific substances to come or some molecules to pass through, Cholesterol - allows for fluidity of the membrane, Glycoproteins - for cell identification they serve as markers	 1 1 1 1
5	a		Pancreas	1
	b		3 from Pancreas detects change Insulin secreted By alpha cells Respiration increased Uptake of glucose increased Liver increases storage of glucose as glycogen	 1 1 1 1 1 1
	c		Any one from: Amount of chocolate, time taken to eat, other food/drink consumed, age, gender, weight, fitness level/metabolic rate, health/pre existing conditions, use of medicines/drugs	 1
	d		Any three from Data suggests that blood glucose returns to normal Doesn't show how much exercise has been done Doesn't say age/gender/other named variable May only be true for chocolate/only one type of food investigated	 1 1 1 1
6			Top left: transpiration increases when wind speed increases/there is a positive correlation Top right: rate increases with pH until the optimum is reached, after the optimum, rate decreases Bottom left: Increasing light initially increases the rate of photosynthesis, but after a while remains constant Bottom right: Population increases slowly at first and then increases at a greater rate/increases exponentially	1 1 1 1

Welcome to Business A Level Business

AS/A-Level Course Outline

Reading and Investigation project

An important part of A Level Business is being able to apply your subject knowledge to different businesses in different situations. Read the case study on New Look and answer the questions below.

New Look slashes prices amid sharp fall in annual sales

Fashion chain New Look is continuing to cut prices as it tries to turn around its business.

New Look wants 80% of its clothes to sell for less than £20.
The price cuts come amid falling sales. Like-for-like sales plunged by 11.7% in the financial year which ended in March, and website sales tumbled 19%.

New Look is one of many retailers this year that struck a Company Voluntary Agreement (CVA) under which a company buys time to sort out its debts.
It is trying to broaden its appeal to include older customers, giving it an age target range of between 18 and 45.

Results from New Look, which has hundreds of stores and has been a High Street presence since 1969, contrast sharply with online rival Boohoo, which also reported results.
Sales of its three brands, Boohoo, PrettyLittleThing and Nasty Gal were 53% higher in the most recent quarter compared with the same quarter last year.

Boohoo brands target customers aged between 18 and 28, with clothes priced at about £15.

New Look vs Boohoo
By Karen Hoggan, business reporter

What's behind the stark contrast in the fortunes of New Look and Boohoo? Is it just that New Look is saddled with an expensive High Street presence, while Boohoo is online only?
Actually it seems to go deeper than that. New Look's online business had a bad year as well.

Charlotte Pearce, retail analyst at GlobalData, says New Look's poor performance is a result of its "loss of relevance" among UK shoppers over the last couple of years.

New Look is broadly targeting the 16-44 year old shopper, while Boohoo and its other brands PrettyLittleThing and Nasty Gal are all much more narrowly focused on 16-24 year olds. By trying to appeal to too many different shoppers, New Look has ended up appealing to none as "its proposition is unclear", says Ms Peace.
She also says New Look's product ranges "lack excitement and its product is much safer" compared with the "daring" designs of the online-only fashion retailers.
Retailers like Boohoo are "more in tune with millennials in terms of providing trend-led and boundary-pushing fast fashion," she says.

As a result she reckons New Look would "struggle" to lure back these 16-24 [year old] shoppers because it hasn't kept up with the likes of Boohoo.

However, Ms Pearce doesn't think New Look should be putting effort into trying to recapture the younger shoppers who have been lured away by Boohoo and others because the competition is so "intense".

Instead she says New Look needs to adjust its target customer base to focus on the older 25-34 year olds.

Clothing retailer Ted Baker also released profits on Tuesday. Its half-year sales were up 4%, mainly thanks to growth in its online sales.

New Look also booked a one-off cost of £34m, partly for discounting old stock. The company said it had made "significant progress", which will be reflected in next year's results.

Last November it brought back Alistair McGeorge, who ran the business between 2012 and 2014. Mr McGeorge said: "Last year was undoubtedly very difficult for New Look, with a well-documented combination of external and self-inflicted issues impacting our performance.

"We still have more work to do to restore long-term profitability, but I am confident we are now better placed to achieve this than we were when I returned to the business over six months ago."

Under its turnaround plan, the company is cutting 1,000 jobs and closing 60 stores. The plan will cut the fashion chain's rents by between 15% to 55% across its remaining 393 stores. Last month, the company hit the headlines with news it was attaching higher prices to its bigger sizes, a policy it said it would reverse.

Adapted from: https://www.bbc.co.uk/news/business-44451139

Questions to consider:

1. Summarise the case study in less than 30 words.

2. What are the key reasons why New Look is struggling in the clothing market?

3. Do you think that New Look have made the right decision by reducing their prices to try and compete with Boohoo and PLT? Explain your answer.

4. What else could New Look do to compete with these other businesses? (Try to think of 2/3 ways)

STRIVE: Of the suggestions you have made in question 4, which do you think would be most effective? Explain your answer.

Developing your business vocabulary

You are going to learn a lot of new vocabulary over the next two years, so here are the core concepts that we would like you to find out about.

Find and learn the definitions of these top 10 business terms using the (fabulous!) website https://www.tutor2u.net/business , you may use examples if it helps to illustrate your point:

1. Trade off

2. Opportunity Cost

3. Added Value

4. Demand

5. Profit

6. Cash Flow

7. Elasticity

8. Motivation

9. Competition

10. Objectives

11. Risk

12. Entrepreneurship

Developing your analysis and evaluation skills

Analysis and evaluation are two key assessment objectives that you will be tested on during your A Level. You will need to prepare a presentation (with no more than 10 slides) which answers questions below; we expect you to do some research and use real life business scenarios to support your arguments.

Remember that the slideshow does not need to contain all of your argument - only key points, which you could elaborate on if you are asked about it. There is no 'right' answer to the statements we have used; we are just trying to get you thinking about the topics you will be studying.

Theme One Topic

"The best way to increase profit is to advertise"

Discuss the extent to which this statement is true, showing a balanced argument and an overall conclusion.

Suggested points:

- What is profit?
- What are the ways which a business can improve profit (including advertisement)? - What evidence have you gathered to support your argument?
- What evidence could be used against your argument?
- Give an overall conclusion about what you think based on your evidence.

OR

Theme Two Topic

"I don't need a plan, I've got a great business idea, I'm just going to run with it"

Discuss the extent to which you agree with the above statement showing a balanced argument and an overall conclusion.

Suggested points:

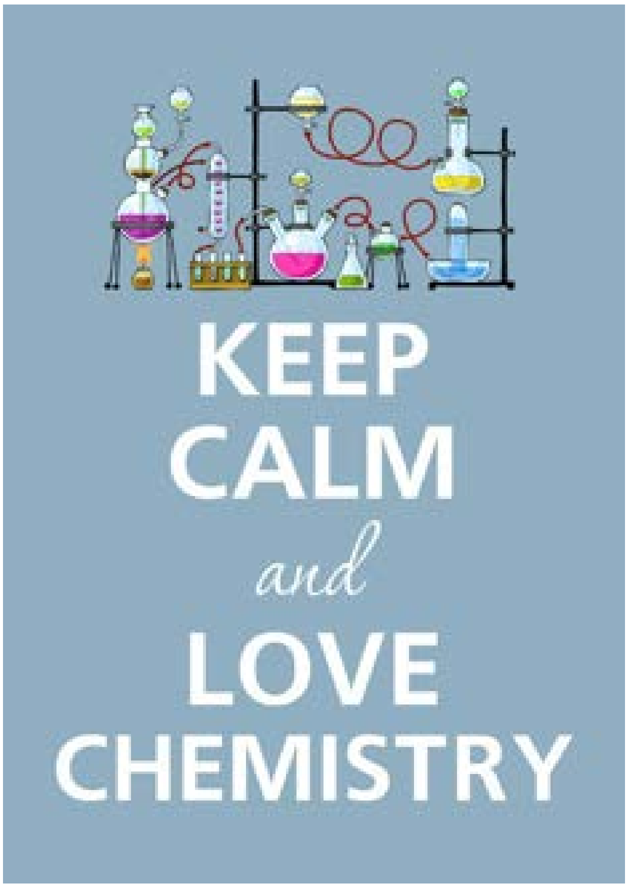
- What is business planning?
- Why is it important / unimportant to have a plan?
- Could you realistically create a business without a plan? How do you know? What evidence do you have? - Give an overall conclusion about what you think based on your evidence.

Transition Pack for A Level Chemistry

Get ready for A-level!
A guide to help you get ready for A-level Chemistry, including everything from topic guides to days out and online learning courses.

Please note: these resources are non-board specific. Please direct your students to the specifics of where this knowledge and skills most apply.

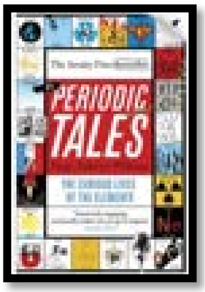
So you are considering A Level Chemistry?



This pack contains a programme of activities and resources to prepare you to start an A level in Chemistry in September. It is aimed to be used after you complete your GCSE, throughout the remainder of the summer term and over the Summer Holidays to ensure you are ready to start your course in September.

Book Recommendations

Periodic Tales: The Curious Lives of the Elements (Paperback) Hugh Aldersey-Williams

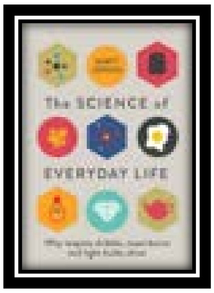


ISBN-10: 0141041455

http://bit.ly/pixlchembook1

This book covers the chemical elements, where they come from and how they are used. There are loads of fascinating insights into uses for chemicals you would have never even thought about.

The Science of Everyday Life: Why Teapots Dribble, Toast Burns and Light Bulbs Shine (Hardback) Marty Jopson

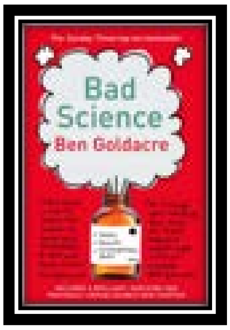


ISBN-10: 1782434186

http://bit.ly/pixlchembook2

The title says it all really, lots of interesting stuff about the things around you home!

Bad Science (Paperback) Ben Goldacre

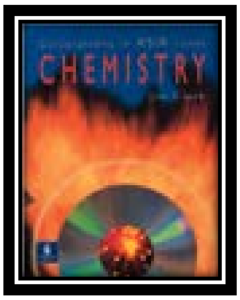


ISBN-10: 000728487X

http://bit.ly/pixlchembook3

Here Ben Goldacre takes apart anyone who published bad / misleading or dodgy science – this book will make you think about everything the advertising industry tries to sell you by making it sound 'sciency'.

Calculations in AS/A Level Chemistry (Paperback) Jim Clark



ISBN-10: 0582411270

http://bit.ly/pixlchembook4

If you struggle with the calculations side of chemistry, this is the book for you. Covers all the possible calculations you are ever likely to come across. Brought to you by the same guy who wrote the excellent chemguide.co.uk website.

Salters' Advanced Chemistry: Chemical Storylines

Do not feel you need to buy the latest edition (unless you are doing Salters chemistry!) You can pick up an old edition for a few pounds on ebay, gives you a real insight into how chemistry is used to solve everyday problems from global pollution through feeding to world to making new medicines to treat disease.

Videos to watch online

Rough science – the Open University – 34 episodes available

Real scientists are 'stranded' on an island and are given scientific problems to solve using only what they can find on the island.

Great fun if you like to see how science is used in solving problems.

There are six series in total

http://bit.ly/pixlchemvid1a

http://www.dailymotion.com/playlist/x2igjq_Rough-Science_rough-science-full-series/1#video=xxw6pr

or

http://bit.ly/pixlchemvid1b

https://www.youtube.com/watch?v=lUoDWAt259I

A thread of quicksilver – The Open University

A brilliant history of the most mysterious of elements – mercury. This program shows you how a single substance led to empires and war, as well as showing you come of the cooler properties of mercury.

http://bit.ly/pixlchemvid2

https://www.youtube.com/watch?v=t46lvTxHHTA

10 weird and wonderful chemical reactions

10 good demonstration reactions, can you work out the chemistry of …. any… of them?

http://bit.ly/pixlchemvid3

https://www.youtube.com/watch?v=0Bt6RPP2ANI

Chemistry in the Movies

Dantes Peak 1997: Volcano disaster movie.

Use the link to look at the Science of acids and how this links to the movie.
http://www.open.edu/openlearn/science-maths-technology/science/chemistry/dantes-peak

http://www.flickclip.com/flicks/dantespeak1.html

http://www.flickclip.com/flicks/dantespeak5.html

Fantastic 4 2005 &2015: Superhero movie

Michio Kaku explains the "real" science behind fantastic four http://nerdist.com/michio-kaku-explains-the-real-science-behind-fantastic-four/

http://www.flickclip.com/flicks/fantastic4.html

Research activities

Use your online searching abilities to see if you can find out as much about the topic as you can. Remember it you are a prospective A level chemist, you should aim to push **your** knowledge.

You can make a 1-page summary for each one you research using Cornell notes:

http://coe.jmu.edu/learningtoolbox/cornellnotes.html

Task 1: The chemistry of fireworks

What are the component parts of fireworks? What chemical compounds cause fireworks to explode? What chemical compounds are responsible for the colour of fireworks?

Task 2: Why is copper sulfate blue?

Copper compounds like many of the transition metal compounds have got vivid and distinctive colours – but why?

Task 3: Aspirin

What was the history of the discovery of aspirin, how do we manufacture aspirin in a modern chemical process?

Task 4: The hole in the ozone layer

Why did we get a hole in the ozone layer? What chemicals were responsible for it? Why were we producing so many of these chemicals? What is the chemistry behind the ozone destruction?

Task 5: ITO and the future of touch screen devices

ITO – indium tin oxide is the main component of touch screen in phones and tablets. The element indium is a rare element and we are rapidly running out of it. Chemists are desperately trying to find a more readily available replacement for it. What advances have chemists made in finding a replacement for it?

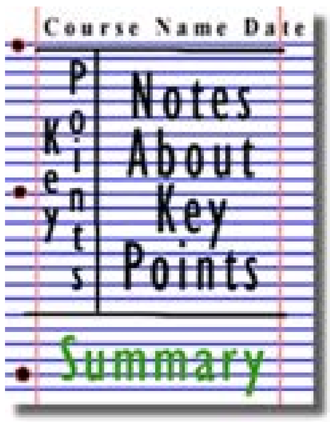


Figure 1: http://coe.jmu.edu/learningtoolbox/images/noteb4.gif

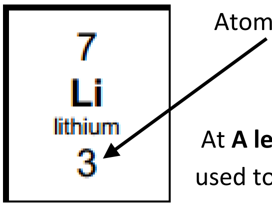
Pre-Knowledge Topics

Chemistry topic 1 – Electronic structure, how electrons are arranged around the nucleus

A periodic table can give you the proton / atomic number of an element, this also tells you how many electrons are in the ***atom***.

You will have used the rule of electrons shell filling, where:

The first shell holds up to 2 electrons, the second up to 8, the third up to 8 and the fourth up to 18 (or you may have been told 8).



Atomic number =3, electrons = 3, arrangement 2 in the first shell and 1 in the second or

Li = 2,1

At **A level** you will learn that the electron structure is more complex than this, and can be used to explain a lot of the chemical properties of elements.

The ‘shells’ can be broken down into ‘orbitals’, which are given letters:’s’ orbitals, ‘p’ orbitals and ‘d’ orbitals.

You can read about orbitals here:

http://bit.ly/pixlchem1

http://www.chemguide.co.uk/atoms/properties/atomorbs.html#top

Now that you are familiar with s, p and d orbitals try these problems, write your answer in the format:

$1s^2$, $2s^2$, $2p^6$ etc.

Q1.1 Write out the electron configuration of:

a) Ca b) Al c) S d) Cl e) Ar f) Fe g) V h) Ni i) Cu j) Zn k) As

Q1.2 Extension question, can you write out the electron arrangement of the following ***ions***:

a) K^+ b) O^{2-} c) Zn^{2+} d) V^{5+} e) Co^{2+}

Chemistry topic 2 – Oxidation and reduction

At GCSE you know that oxidation is adding oxygen to an atom or molecule and that reduction is removing oxygen, or that oxidation is removing hydrogen and reduction is adding hydrogen. You may have also learned that oxidation is removing electrons and reduction is adding electrons.

At A level we use the idea of ***oxidation number*** a lot!

You know that the metals in group 1 react to form ions that are +1, i.e. Na^+ and that group 7, the halogens, form -1 ions, i.e. Br -.

We say that sodium, when it has reacted has an oxidation number of +1 and that bromide has an oxidation number of -1.

All atoms that are involved in a reaction can be given an oxidation number.

An element, Na or O_2 is always given an oxidation state of zero (0), any element that has reacted has an oxidation state of + or -.

As removing electrons is **reduction**, if, in a reaction the element becomes **more** negative it has been reduced, if it becomes more positive it has been oxidised.

-5 0 +5

You can read about the rules for assigning oxidation numbers here:

http://www.dummies.com/how-to/content/rules-for-assigning-oxidation-numbers-to-elements.html

Elements that you expect to have a specific oxidation state actually have different states, so for example you would expect chlorine to be -1, it can have many oxidation states: NaClO, in this compound it has an oxidation state of +1

There are a few simple rules to remember:

Metals have a + oxidation state when they react.

Oxygen is 'king' it always has an oxidation state of -2

Hydrogen has an oxidation state of +1 (except metal hydrides)

The charges in a molecule must cancel.

Examples: Sodium nitrate, $NaNO_3$			sulfate ion, SO_4^{2-}	
	Na +1	3x O^{2-}	$4xO^{2-}$	and 2- charges 'showing'
	+1	-6	-8	-2
To cancel:	N = +5		S = +6	

Q2.1 Work out the oxidation state of the **underlined** atom in the following:

a) $Mg\underline{C}O_3$ b) $\underline{S}O_3$ c) $Na\underline{Cl}O_3$ d) $\underline{Mn}O_2$ e) $\underline{Fe}_2O_3$ f) $\underline{V}_2O_5$

g) $K\underline{Mn}O_4$ h) $\underline{Cr}_2O_7^{2-}$ i) $\underline{Cl}_2O_4$

Chemistry topic 3 – Isotopes and mass

You will remember that an isotopes are elements that have differing numbers of neutrons. Hydrogen has 3 isotopes; H_1^1 H_1^2 H_1^3

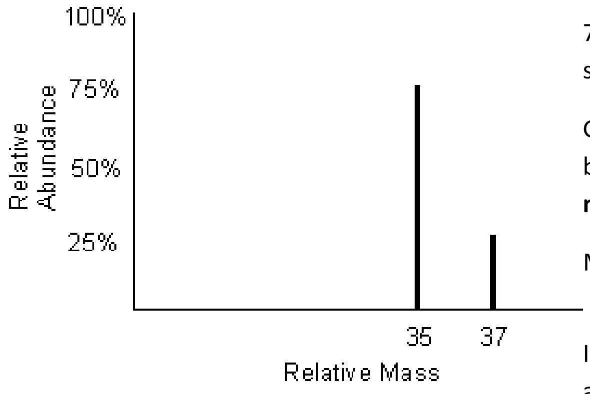
Isotopes occur naturally, so in a sample of an element you will have a mixture of these isotopes. We can accurately measure the amount of an isotope using a **mass spectrometer**. You will need to understand what a mass spectrometer is and how it works at A level. You can read about a mass spectrometer here:

http://bit.ly/pixlchem3
http://www.kore.co.uk/tutorial.htm
http://bit.ly/pixlchem4
http://filestore.aqa.org.uk/resources/chemistry/AQA-7404-7405-TN-MASS-SPECTROMETRY.PDF

Q3.1 What must happen to the atoms before they are accelerated in the mass spectrometer?

Q3.2 Explain why the different isotopes travel at different speeds in a mass spectrometer.

A mass spectrum for the element chlorine will give a spectrum like this:



75% of the sample consist of chlorine-35, and 25% of the sample is chlorine-37.

Given a sample of naturally occurring chlorine ¾ of it will be Cl-35 and ¼ of it is Cl-37. We can calculate what the **mean** mass of the sample will be:

$$\text{Mean mass} = \frac{75}{100} \times 35 + \frac{25}{100} \times 37 = 35.5$$

If you look at a periodic table this is why chlorine has an atomic mass of 35.5.

http://www.avogadro.co.uk/definitions/ar.htm

An A level periodic table has the masses of elements recorded much more accurately than at GCSE. Most elements have isotopes and these have been recorded using mass spectrometers.

GCSE

11 B boron 5	12 C carbon 6	14 N nitrogen 7	16 O oxygen 8	19 F fluorine 9
27 Al aluminium 13	28 Si silicon 14	31 P phosphorus 15	32 S sulfur 16	35.5 Cl chlorine 17

A level

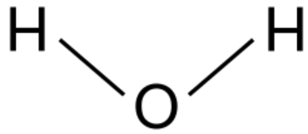
10.8 B 5 boron	12.0 C 6 carbon	14.0 N 7 nitrogen	16.0 O 8 oxygen	19.0 F 9 fluorine
27.0 Al 13 aluminium	28.1 Si 14 silicon	31.0 P 15 phosphorus	32.1 S 16 sulphur	35.5 Cl 17 chlorine

Given the percentage of each isotope you can calculate the mean mass which is the accurate atomic mass for that element.

Q3.3 Use the percentages of each isotope to calculate the accurate atomic mass of the following elements.

a) Antimony has 2 isotopes: Sb-121 57.25% and Sb-123 42.75%
b) Gallium has 2 isotopes: Ga-69 60.2% and Ga-71 39.8%
c) Silver has 2 isotopes: Ag-107 51.35% and Ag-109 48.65%
d) Thallium has 2 isotopes: Tl-203 29.5% and Tl-205 70.5%
e) Strontium has **4** isotopes: Sr-84 0.56%, Sr-86 9.86%, Sr-87 7.02% and Sr-88 82.56%

Chemistry topic 4 – The shapes of molecules and bonding.



Have you ever wondered why your teacher drew a water molecule like this?

The lines represent a covalent bond, but why draw them at an unusual angle?

If you are unsure about covalent bonding, read about it here:

http://bit.ly/pixlchem5

http://www.chemguide.co.uk/atoms/bonding/covalent.html#top

At A level you are also expected to know how molecules have certain shapes and why they are the shape they are.

You can read about shapes of molecules here:

http://bit.ly/pixlchem6

http://www.chemguide.co.uk/atoms/bonding/shapes.html#top

Q4.1 Draw a dot and cross diagram to show the bonding in a molecule of aluminium chloride ($AlCl_3$)

Q4.2 Draw a dot and cross diagram to show the bonding in a molecule of ammonia (NH_3)

Q4.3 What is the shape and the bond angles in a molecule of methane (CH_4)?

Chemistry topic 5 – Chemical equations

Balancing chemical equations is the stepping stone to using equations to calculate masses in chemistry.

There are loads of websites that give ways of balancing equations and lots of exercises in balancing.

Some of the equations to balance may involve strange chemical, don't worry about that, the key idea is to get balancing right.

http://bit.ly/pixlchem7

http://www.chemteam.info/Equations/Balance-Equation.html

This website has a download; it is safe to do so:

http://bit.ly/pixlchem8

https://phet.colorado.edu/en/simulation/balancing-chemical-equations

Q5.1 Balance the following equations

a. $H_2 + 0_2 \rightarrow H_20$

b. $S_8 + 02 \rightarrow S0_3$

c. $HgO \rightarrow Hg + 0_2$

d. $Zn + HCl \rightarrow ZnCl_2 + H_2$

e. Na+ H_20 → NaOH + H_2

f. $C_{10}H_{16}$+ Cl_2 → C + HCl

g. Fe+ 0_2→ Fe_20_3

h. $C_6H_{12}0_6$+ 0_2→ $C0_2$+ H_20

i. Fe_20_3 + H_2 → Fe + H_20

j. Al + FeO → Al_2O_3 + Fe

Chemistry topic 6 – Measuring chemicals – the mole

From this point on you need to be using an A level periodic table, not a GCSE one you can view one here:

http://bit.ly/pixlpertab

https://secondaryscience4all.files.wordpress.com/2014/08/filestore_aqa_org_uk_subjects_aqa-2420-w-trb-ptds_pdf.png

Now that we have our chemical equations balanced, we need to be able to use them in order to work out masses of chemicals we need or we can produce.

The ***mole*** is the chemists equivalent of a dozen, atoms are so small that we cannot count them out individually, we weigh out chemicals.

For example: magnesium + sulfur → magnesium sulfide

Mg + S → MgS

We can see that one atom of magnesium will react with one atom of sulfur, if we had to weigh out the atoms we need to know how heavy each atom is.

From the periodic table: Mg = 24.3 and S = 32.1

If I weigh out exactly 24.3g of magnesium this will be 1 mole of magnesium, if we counted how many atoms were present in this mass it would be a huge number (6.02×10^{23}!!!!), if I weigh out 32.1g of sulfur then I would have 1 mole of sulfur atoms.

So 24.3g of Mg will react precisely with 32.1g of sulfur, and will make 56.4g of magnesium sulfide.

Here is a comprehensive page on measuring moles, there are a number of descriptions, videos and practice problems.

You will find the first 6 tutorials of most use here, and problem sets 1 to 3.

http://bit.ly/pixlchem9

http://www.chemteam.info/Mole/Mole.html

Q6.1 Answer the following questions on moles.

a) How many moles of phosphorus pentoxide (P_4O_{10}) are in 85.2g?
b) How many moles of potassium in 73.56g of potassium chlorate (V) ($KClO_3$)?

c) How many moles of water are in 249.6g of hydrated copper sulfate(VI) ($CuSO_4.5H_2O$)? For this one, you need to be aware the dot followed by $5H_2O$ means that the molecule comes with 5 water molecules so these have to be counted in as part of the molecules mass.

d) What is the mass of 0.125 moles of tin sulfate ($SnSO_4$)?

e) If I have 2.4g of magnesium, how many g of oxygen(O_2) will I need to react completely with the magnesium? $2Mg + O_2 \rightarrow MgO$

Chemistry topic 7 – Solutions and concentrations

In chemistry a lot of the reactions we carry out involve mixing solutions rather than solids, gases or liquids.

You will have used bottles of acids in science that have labels saying 'Hydrochloric acid 1M', this is a solution of hydrochloric acid where 1 mole of HCl, hydrogen chloride (a gas) has been dissolved in $1dm^3$ of water.

The dm^3 is a cubic decimetre, it is actually 1 litre, but from this point on as an A level chemist you will use the dm^3 as your volume measurement.

http://bit.ly/pixlchem10

http://www.docbrown.info/page04/4_73calcs11msc.htm

Q7.1

a) What is the concentration (in mol dm^{-3}) of 9.53g of magnesium chloride ($MgCl_2$) dissolved in $100cm^3$ of water?

b) What is the concentration (in mol dm^{-3}) of 13.248g of lead nitrate ($Pb(NO_3)_2$) dissolved in $2dm^3$ of water?

c) If I add $100cm^3$ of 1.00 mol dm^3 HCl to $1.9dm^3$ of water, what is the molarity of the new solution?

d) What mass of silver is present in $100cm^3$ of $1moldm^{-3}$ silver nitrate ($AgNO_3$)?

e) The Dead Sea, between Jordan and Israel, contains 0.0526 $moldm^{-3}$ of Bromide ions (Br^-), what mass of bromine is in $1dm^3$ of Dead Sea water?

Chemistry topic 8 – Titrations

One key skill in A level chemistry is the ability to carry out accurate titrations, you may well have carried out a titration at GCSE, at A level you will have to carry them out very precisely **and** be able to describe in detail how to carry out a titration - there will be questions on the exam paper about how to carry out practical procedures.

You can read about how to carry out a titration here, the next page in the series (page 5) describes how to work out the concentration of the unknown.

http://bit.ly/pixlchem11

http://www.bbc.co.uk/schools/gcsebitesize/science/triple_aqa/further_analysis/analysing_substances/revision/4/

Remember for any titration calculation you need to have a balanced symbol equation; this will tell you the ratio in which the chemicals react.

E.g. a titration of an unknown sample of sulfuric acid with sodium hydroxide.

A $25.00 cm^3$ sample of the unknown sulfuric acid was titrated with $0.100 moldm^{-3}$ sodium hydroxide and required exactly $27.40 cm^3$ for neutralisation. What is the concentration of the sulfuric acid?

Step 1: the equation $2NaOH + H_2SO_4 \rightarrow Na_2SO_4 + 2H_2O$

Step 2; the ratios 2 : 1

Step 3: how many moles of sodium hydroxide $27.40 cm^3 = 0.0274 dm^3$

number of moles = c x v = 0.100 x 0.0274 = 0.00274 moles

step 4: Using the ratio, how many moles of sulfuric acid

for every 2 NaOH there are 1 H_2SO_4 so, we must have 0.00274/2 =0.00137 moles of H_2SO_4

Step 5: Calculate concentration. concentration = moles/volume $\leftarrow$in dm^3 = 0.00137/0.025 = ***0.0548 moldm^{-3}***

Here are some additional problems, which are harder, ignore the questions about colour changes of indicators.

http://bit.ly/pixlchem12

http://www.docbrown.info/page06/Mtestsnotes/ExtraVolCalcs1.htm

Use the steps on the last page to help you

Q8.1 A solution of barium nitrate will react with a solution of sodium sulfate to produce a precipitate of barium sulfate.

$Ba(NO_3)_2(aq) + Na_2SO_4(aq) \rightarrow BaSO_4(s) + 2NaNO_3(aq)$

What volume of $0.25 moldm^{-3}$sodium sulfate solution would be needed to precipitate all of the barium from $12.5 cm^3$ of $0.15 moldm^{-3}$ barium nitrate?

Chemistry topic 9 – Organic chemistry – functional groups

At GCSE you would have come across **hydrocarbons** such as alkanes (ethane etc) and alkenes (ethene etc). You may have come across molecules such as alcohols and carboxylic acids. At A level you will learn about a wide range of molecules that have had atoms added to the carbon chain. These are called functional groups, they give the molecule certain physical and chemical properties that can make them incredibly useful to us.

Here you are going to meet a selection of the functional groups, learn a little about their properties and how we give them logical names.

You will find a menu for organic compounds here:

http://bit.ly/pixlchem13

http://www.chemguide.co.uk/orgpropsmenu.html#top

And how to name organic compounds here:

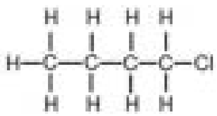
http://bit.ly/pixlchem14

http://www.chemguide.co.uk/basicorg/conventions/names.html#top

Using the two links see if you can answer the following questions:

Q9.1 Halogenoalkanes

What is the name of this halogenoalkane?



How could you make it from butan-1-ol?

Q9.2 Alcohols

How could you make ethanol from ethene?

How does ethanol react with sodium, in what ways is this a) similar to the reaction with water, b) different to the reaction with water?

Q9.3 Aldehydes and ketones

Draw the structures of a) propanal b) propanone

How are these two functional groups different?

Chemistry topic 10 – Acids, bases, pH

At GCSE you will know that an acid can dissolve in water to produce H^+ ions, at A level you will need a greater understanding of what an acid or a base is.

Read the following page and answer the questions

http://bit.ly/pixlchem15

http://www.chemguide.co.uk/physical/acidbaseeqia/theories.html#top

Q10.1 What is your new definition of what an acid is?

Q10.2 How does ammonia (NH_3) act as a base?

http://bit.ly/pixlchem16

http://www.chemguide.co.uk/physical/acidbaseeqia/acids.html#top

Q10.3 Ethanoic acid (vinegar) is a weak acid, what does this mean?

Q10.4 What is the pH of a solution of 0.01 $moldm^{-3}$ of the strong acid, hydrochloric acid?

Pre-Knowledge Topics Answers to problems

Q1.1a) $1s^2\ 2s^2\ 2p^63s^23p^64s^2$ b) $1s^2\ 2s^2\ 2p^63s^23p^1$ c) $1s^2\ 2s^2\ 2p^63s^23p^4$ d) $1s^2\ 2s^2\ 2p^63s^23p^5$

e) $1s^2\ 2s^2\ 2p^63s^23p^6$ f) $1s^2\ 2s^2\ 2p^63s^23p^6\ 3d^64s^2$ g) $1s^2\ 2s^2\ 2p^63s^23p^6\ 3d^34s^2$

h) $1s^2\ 2s^2\ 2p^63s^23p^6\ 3d^84s^2$ i) $1s^2\ 2s^2\ 2p^63s^23p^6\ 3d^{10}\ 4s^1$ j) $1s^2\ 2s^2\ 2p^63s^23p^6\ 3d^{10}\ 4s^2$

k) $1s^2\ 2s^2\ 2p^63s^23p^6\ 4s^2\ 3d^{10}\ 4p^3$

Q1.2a) $1s^2\ 2s^2\ 2p^63s^23p^6$ b) $1s^2\ 2s^2\ 2p^63s^23p^6$ c) $1s^2\ 2s^2\ 2p^63s^23p^6\ 3d^{10}$

d) $1s^2\ 2s^2\ 2p^63s^23p^6$ e) $1s^2\ 2s^2\ 2p^63s^23p^6\ 3d^7$

===

Q2.1 a) +4 b) +6 c) +5 d) +4 e) +3 f) +5 g) +7 h) +6 i) +4

===

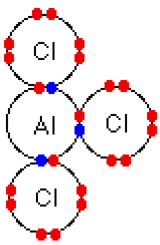
Q3.1 They must be ionised / turned into ions

Q3.2 The ions are all given the same amount of kinetic energy, as KE = ½ mv^2 the lighter ions will have greater speed / heavier ions will have less speed.

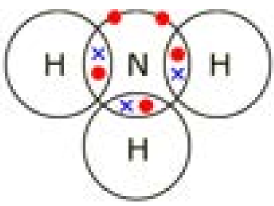
Q3.3 a) 121.855 b) 67.796 c) 107.973 d) 204.41 e) 87.710 / 87.7102

===

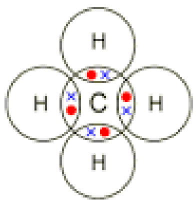
Q4.1

a) 

120°

b) 

107°

c) 

109.5°

===

Q5a. 2H_2 + 0_2 → 2 H_20

b. S_8+ 1202 → 8$S0_3$

c. 2HgO → 2Hg+ 0_2

d. Zn+ 2HCl → $ZnCl_2$+ H_2

e. 2Na+ 2H_20 → 2NaOH + H_2

f. $C_{10}H_{16}$+ 8Cl_2 → 10C + 16HCl

g. 2Fe+ 3$0_2$ → 2Fe_20_3

h. $C_6H_{12}0_6$+ 6$0_2$ → 6 $C0_2$+ 6 H_20

i. Fe_20_3 + 3H_2 → 2Fe + 3H_20

j. 2Al + 3 FeO → Al_2O_3 + 3Fe

Q6.1 a) 85.2/284 = 0.3 moles b) 73.56/122.6 = 0.6 moles c) 249.5/249.5 = 1.0 moles

d) 0.125 x 212.8 = 26.6g   e) 2Mg : 2O or 1:1 ratio   2.4g of Mg = 0.1moles   so we need 0.1 moles of oxygen (O_2): 0.1 x 32 = 3.2g

7.1 a) 9.53g/95.3 = 0.1 moles, in 100cm^3 or 0.1dm^3 in 1dm^3 0.1moles/0.1dm^3 = 1.0 mol dm^{-3}

b) 13.284g/331.2 = 0.04 moles, in 2dm^3 in 1dm^3 0.04moles /2dm^3 = 0.02 mol dm^{-3}

c) 100cm^3 of 0.1 mol dm^{-3} = 0.01 moles added to a total volume of 2 dm^3 = 0.01moles/2dm^3 = 0.005 mol dm^{-3}

d) in 1dm^3 of 1 mol dm^{-3} silver nitrate, 1 mole of Ag = 107.9g in 0.1dm^3 = 107.9 x 0.1 = 10.79g

e) 0.0526 x 79.7 = 42.0274g

==

8.1

$Ba(NO_3)_2$: Na_2SO_4

1 : 1 ratio

12.5cm^3 of $Ba(NO_3)_2$ = 0.0125dm^3

0.15 moldm^{-3} x 0.0125dm^3 = 0.001875 moles

same number of moles of sodium sulfate needed, which has a concentration of 0.25 mol dm^{-3}

0.001875 moles / 0.25 mol dm^{-3} = 0.0075 dm^3 or 7.5cm^3

==

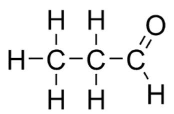
9.1 1-chlorobutane

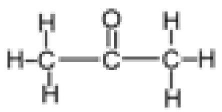
Add butan-1-ol to concentrated HCl and shake

9.2 react ethene with hydrogen gas at high temperature and pressure with a nickel catalyst

The reaction is similar in that it releases hydrogen but different as it proceeds much slower than in water

9.3 propanal   propanone





The carbon atom joined to oxygen in propanal has a hydrogen attached to it, it does not in propanone.

==

10.1 An acid is a proton donor

10.2 Ammonia can accept a proton, to become NH_4^+

10.3 ethanoic acid has not fully dissociated, it has not released all of its hydrogen ions into the solution.

$CH_3COOH \leftrightharpoons CH_3COO^- + H^+$
Mostly this   Very few of these

10.4 pH = -log [0.01] = 2   The pH = 2

Places to visit

1. Go outdoors!
   Have you actually spent any time observing the geology of the area you live in? What rocks or minerals are found in your area? Does your area have a history of extracting minerals? If so what were they, what were they used for, how did they obtain them? Are there any working or remains of mineral extraction industries?

2. Are there any chemical or chemistry based businesses in your area? A big ask, but one that could be really beneficial to you, write them a letter explaining that you are taking A level chemistry and you want to see how chemistry is used in industry and you would like to visit / have some work experience. You never know this could lead to great things!!!!

3. You could also try writing to / searching for your nearest university to see if they are running any summer schools for chemistry – they are usually free and give you the opportunity to experience the laboratories in a university.

4. Science museums.
   You could visit your nearest science museum. They often have special exhibitions that may be of interest to you.
   https://en.wikipedia.org/wiki/List_of_science_museums#United_Kingdom

5. Somerset Earth Science Centre:

   http://www.earthsciencecentre.org.uk

6. The UK Association for Science and Discovery Centres (ASDC)

   This association brings together over 60 major science engagement organisations in the UK.

   http://sciencecentres.org.uk/centres/weblinks.php

Chemistry A level transition - baseline assessment. **40 marks**

All data is given on this paper, you will not need a periodic table

Answer all questions.

1. Here is part of a periodic table, use it to answer the following questions

10.8 B 5 boron	12.0 C 6 carbon	14.0 N 7 nitrogen	16.0 O 8 oxygen	19.0 F 9 fluorine	20.2 Ne 10 neon
27.0 Al 13 aluminium	28.1 Si 14 silicon	31.0 P 15 phosphorus	32.1 S 16 sulphur	35.5 Cl 17 chlorine	39.9 Ar 18 argon

a. Which is the correct electron configuration for a nitrogen atom, circle the correct answer [1]

$1s^22p^5$ $1s^12p^6$ $1s^22s^22p^3$ $1s^22s^5$ $1s^22s^22p^63s^23p^2$

b. Which is the correct electron configuration for a chlorine atom, circle the correct answer [1]

$1s^22s^82p^7$ $1s^22s^22p^82d^5$ $1s^22s^22p^63d^7$ $1s^22s^22p^63p^7$ $1s^22s^22p^63s^23p^5$

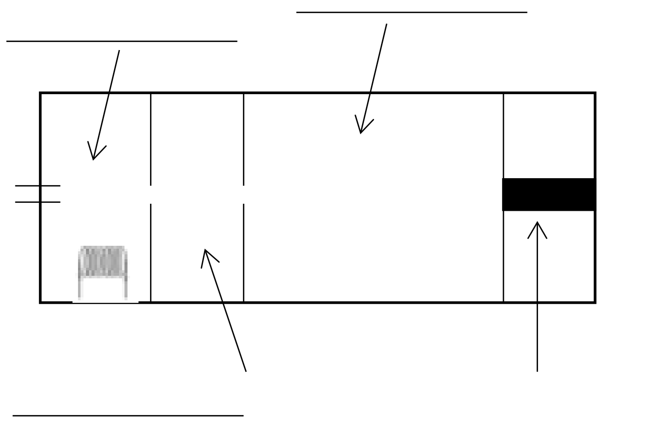
c. Which is the correct electron configuration for an aluminium **ion**, Al^{3+}? Circle the correct answer [1]

$1s^22s^22p^6$ $1s^22s^22p^63s^23p^3$ $1s^22s^22p^63s^2$ $1s^22s^22p^62d^1$

2. Draw a dot and cross diagram to show the bonding in a molecule of water, H_2O. [2]
Atomic numbers: H =1, O =8

3. A time of flight mass spectrometer has 4 main stages. put the correct stage in the diagram below:

Drift region **Ionisation** **Detector** **Acceleration**



[4]

4. A mass spectrometer was used to analyse a sample of chlorine; the results of the analysis are as follows:

isotope mass	% of sample
Cl-35	75.53
Cl-37	24.47

Calculate the accurate atomic mass of chlorine. Give your answer to **3 decimal places**. [3]

mass: ________________________

5. Give the oxidation state of the underlined atom in the following chemicals.
Useful information: H = +1, K = +1, Na = +1, Mg = +2, O = -2, Cl = -1 [7]

a) $\underline{C}O_2$ b) $\underline{S}O_3$ c) $H_2\underline{S}O_4$ d) $\underline{Al}Cl_3$

e) $\underline{Cr}_2O_3$ f) $Na\underline{N}O_3$ g) $\underline{V}Cl_4$

6. Balance the following chemical equations:

a) C_3H_8 + ___ O_2 → ___ CO_2 + ___ H_2O [3]

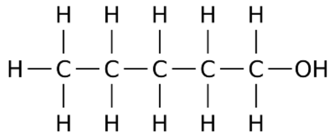
b) ___ HCl + $Mg(OH)_2$ → $MgCl_2$ + ___ H_2O [2]

c) Na_2CO_3 + ___ HCl → ___ NaCl + ___ H_2O + CO_2 [3]

7. Calculate the relative formula masses of the following:
Atomic masses: H = 1, O = 16, S = 32.1, C = 12, Ca = 40.1, Na = 23, Cl = 35.5, Zn = 65.4

a) $CaCl_2$ b) H_2CO_3 c) Na_2SO_4 d) C_3H_7OH e) $Zn(NO_3)_2$ [5]

8. A student carried out a reaction with this molecule:



a. What is the name of this molecule? __________________ [2]

9. Vinegar is a solution of ethanoic acid (CH_3COOH) in water. A student carried out a titration of a sample of vinegar.
He used a pipette to measure exactly 25.0cm^3 of vinegar into a flask, added an indicator and titrated it with a 1.00 mol dm^{-3} solution of sodium hydroxide (NaOH).
The reaction is:

$$CH_3COOH + NaOH \rightarrow CH_3COONa + H_2O$$

The student found that his average titration was 27.50cm^3

c = n/v   c = concentration (mol dm^{-3}), n = number of moles, v = volume (dm^3)

n = m/Rfm   n = number of moles, m = mass in grams, Rfm = formula mass

1dm^3 = 1000 cm^3

a. Using the chemical equation, how many moles of sodium hydroxide will react with 1 mole of ethanoic acid?

___________moles [1]

b. How many moles of sodium hydroxide are in 27.50cm^3 of 1.00 moldm^{-3} sodium hydroxide?

___________moles [2]

c. How many moles of ethanoic acid are in 25.0cm^3 of the vinegar sample?

___________moles [1]

d. How many moles of ethanoic acid are in 1dm^3 of vinegar?

___________moles [1]

e. Ethanoic acid has a formula mass of 48. What mass of ethanoic acid is present in 1dm^3 of vinegar?

___________g [2]

Chemistry A level transition - baseline assessment. - Answers

1. .
   a. Which is the correct electron configuration for a nitrogen atom, circle the correct answer [1]

   $1s^2 2p^5$ $1s^1 2p^6$ $1s^2 2s^2 2p^3$ $1s^2 2s^5$ $1s^2 2s^2 2p^6 3s^2 3p^2$

   b. Which is the correct electron configuration for a chlorine atom, circle the correct answer [1]

   $1s^2 2s^8 2p^7$ $1s^2 2s^2 2p^8 2d^5$ $1s^2 2s^2 2p^6 3d^7$ $1s^2 2s^2 2p^6 3p^7$ $1s^2 2s^2 2p^6 3s^2 3p^5$

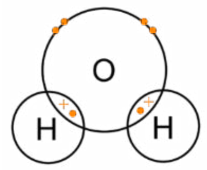
   c. Which is the correct electron configuration for an aluminium **ion**, Al^{3+}? Circle the correct answer [1]

   $1s^2 2s^2 2p^6$ $1s^2 2s^2 2p^6 3s^2 3p^3$ $1s^2 2s^2 2p^6 3s^2$ $1s^2 2s^2 2p^6 2d^1$

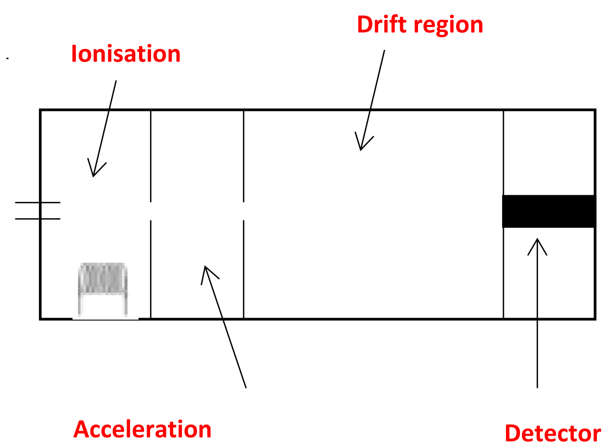
2. Draw a dot and cross diagram to show the bonding in a molecule of water, H_2O. [2]
   Atomic numbers: H =1, O =8

1 mark for 2 x shared electrons

1 mark for lone pairs



3. A time of flight mass spectrometer has 4 main stages. put the correct stage in the diagram below:



[4]

4. A mass spectrometer was used to analyse a sample of chlorine, the results of the analysis are as follows:

isotope mass	% of sample
Cl-35	75.53
Cl-37	24.47

(35x75.53) + (37x24.47)/100 [1]

= 35.4894 [1]

To 3dp = 35.489 [1] [2 marks if above line is missing]

[3]

5. Give the oxidation state of the underlined atom in the following chemicals.
Useful information: H = +1, K = +1, Na = +1, Mg = +2, O = -2, Cl = -1 [7]

a) $\underline{C}O_2$ +4 b) $\underline{S}O_3$ +6 c) $H_2\underline{S}O_4$ +6 d) $\underline{Al}Cl_3$ +3

e) $\underline{Cr}_2O_3$ +3 f) $Na\underline{N}O_3$ +5 g) $\underline{V}Cl_4$ +4

6. Balance the following chemical equations:

a) C_3H_8 + _5_ O_2 → _3_ CO_2 + _4_ H_2O [3]

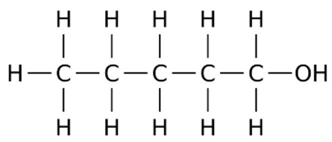
b) _2_ HCl + $Mg(OH)_2$ → $MgCl_2$ + _2_ H_2O [2]

c) Na_2CO_3 + _2_ HCl → _2_ NaCl + _1_ H_2O + CO_2 [3]

7. Calculate the relative formula masses of the following:
Atomic masses: H = 1, O = 16, S = 32.1, C = 12, Ca = 40.1, Na = 23, Cl = 35.5

a) $CaCl_2$	b) H_2CO_3	c) Na_2SO_4	d) C_3H_7OH	e) $Zn(NO_3)_2$	[5]
111.1	62	142.3	60	189.4	

8. A student carried out a reaction with this molecule:



a. What is the name of this molecule? pentan-1-ol [2]

Pentanol = 1 mark pentan-1-ol = 2 marks

9.

a. Using the chemical equation, how many moles of sodium hydroxide will react with 1 mole of ethanoic acid?

______1______moles [1]

b. How many moles of sodium hydroxide are in 27.50cm^3 of 1.00 moldm^{-3} sodium hydroxide?

27.5/1000 [1] x 1.00 = 0.0275 [1]

0.0275 [2] moles [2]

c. How many moles of ethanoic acid are in 25.0cm^3 of the vinegar sample?

___0.0275 __moles [1]

d. How many moles of ethanoic acid are in $1dm^3$ of vinegar?

0.0275 x 1000/25 = 1.10

___1.10____moles [1]

e. Ethanoic acid has a formula mass of 48. What mass of ethanoic acid is present in $1dm^3$ of vinegar?

1.1 x 48 = 52.8g

___52.8g ___g [1]

Criminology Bridging Task

Task 1: You will cover a range of crimes that we look at in more depth on the syllabus.

<u>White Collar Crime</u>
Example - Bernie Madoff

<u>Technological Crime</u>
Example - Gary McKinnon

<u>Hate Crimes</u>
Example - Matthew Shepherd

<u>Honour Crimes</u>
Example - Shafilea Ahmed

<u>Domestic Abuse</u>
Example - Alex Skeel or Tina Nash

You need to create a table, which addresses the following points.

- Define this type of crime
- Explain the laws regarding this type of crime
- Outline a famous case
  - a. Who was/were the victim(s)?
  - b. Who was/were the perpetrator(s)?
  - c. What happened to the victim?
  - d. What was the motivation for the crime?
  - e. What was the outcome of any court case or trial?

Task 2: Research three different explanations of crime and write 500 words on each.

1. Biological Theories of Crime
2. Individualistic Theories of Crime
3. Sociological Theories of Crime

Biological theories of crime could include one, or both of: Lombroso theory or genetics.

Individualistic theories of crime could include one or both of: Bandura's learning theory, Freud's psychodynamic theory of crime

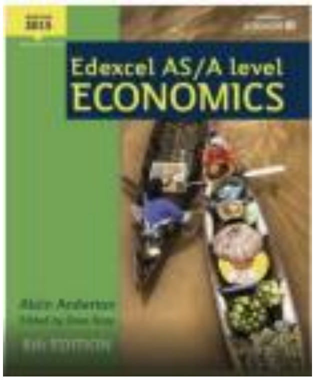
Sociological theories of crime could include one, or both of: Marxism, Labelling Theory, Left and Right realism.

A-Level Economics Summer Bridging Work

Exam board: Edexcel - Economics A

Resources that you need to purchase in preparation for studying this course:

• Course textbook: “Pearson Edexcel A Level Economics A Fourth Edition”



• Lever arch folder
• A copy of the course specification (found here: https://qualifications.pearson.com/content/dam/pdf/A%20Level/Economics/2015/specification-and-sample-assessmentmaterials/A_Level_Econ_A_Spec.pdf) •
Mini white board (will be given in class but a good idea to have one of your own) •
White board pens
• Calculator
• Plastic wallets
• Line paper

Macro Economics Tasks

Task 1 An introduction to macroeconomics: Venezuela

We will spend a lot of time in lessons discussing examples from around the world. We will try to apply economic theory to explain and analyse what we see in both the UK and internationally.

Macroeconomics is the study of the economy as a whole and covers many of the concepts you might hear regularly on the news such as unemployment, inflation, trade and recessions.

For this session, I would like you to consider the economy of Venezuela – it is a truly fascinating example of a country that should be far richer than it actually is. I think it’s a great case study to get you started on the wide range of topics we cover in economics – I hope you find it interesting too.
Most of the questions relate to the BBC video in the link below. You do not need to print the worksheet out. You can type your answers directly underneath each question or you could

handwrite if you prefer.

Remember you are just starting out on your Economics journey: I am not expecting you to know the answers to everything, approach the questions with an inquiring mind and see where you it takes you!

Watch the entire programme, you are likely to need to pause, rewind and perhaps watch some parts more than once. BBC Caribbean with Simon Reeve episode 2

1. Identify 2 reasons why Venezuela should be 'one of the world's richest countries'? 2.

Extension: Which other countries have large oil reserves? Think before you click!

map countries most oil reserves How does each country's GDP compare?
How to economists make use of GDP data? Watch this video Changing world economy - use of GDP data

Where does Venezuela rank in GDP/capita data? Do any countries have higher or lower GDP/capita than you were expecting?

https://www.usatoday.com/story/money/2019/05/22/largest-oil-reserves-inworld-15- countries-that-control-the-worlds-oil/39497945/

3. Before you continue watching the BBC video, think of reasons WHY Venezuela's GDP per capita might not be as high as expected (given the oil reserves)? You will find out some possible answers as you watch the video.

-
-
-

4. Throughout the video, there are lots of examples of Venezuela's worsening living standards. You can come back to this question as you go along. Applying your knowledge is a key skill for economics students.

- 1 in 3 Venezuelans are poor
-
-
-
-
-

5. Simon comments that 'wealth is in the hands of the few in Venezuela'. Watch how economists measure inequality using Lorenz curves/Gini coefficient'

Lorenz curve and Gini coefficient tutorial

Think about what it means for a country to have high levels of inequality and the impact it has on society. Is inequality inevitable in (free market) economies?

6. The reasons given for Venezuela's economic decline are given as 'restrictions on foreign currency meaning that businesses don't want to import and 'Soviet-style' controls on prices meaning that shops don't want to sell.

Extension question: Read/watch the explanations about the objectives of producers / functions of the price mechanism. . Explain why Venezuelan shops might be behaving rationally if prices are set below the free market equilibrium.

7. How much does it cost to fill a tank of petrol? How does this compare to your local petrol station? What reasons are given for why petrol is so cheap?

8. How does this government policy (in question 8) lead to a more unequal distribution of income in Venezuela?

9. Simon visits a family living with an oil pump outside their house. What problems have the family suffered/witnessed as a result?

10. Petrol in Columbia costs 100 times the price in Venezuela and large numbers of people illegally smuggle petrol at the border.

Explain the impact of this smuggling on the Venezuelan economy. Consider the impact of the lost tax revenue? How does it change the incentives for the taxi driver to seek work?

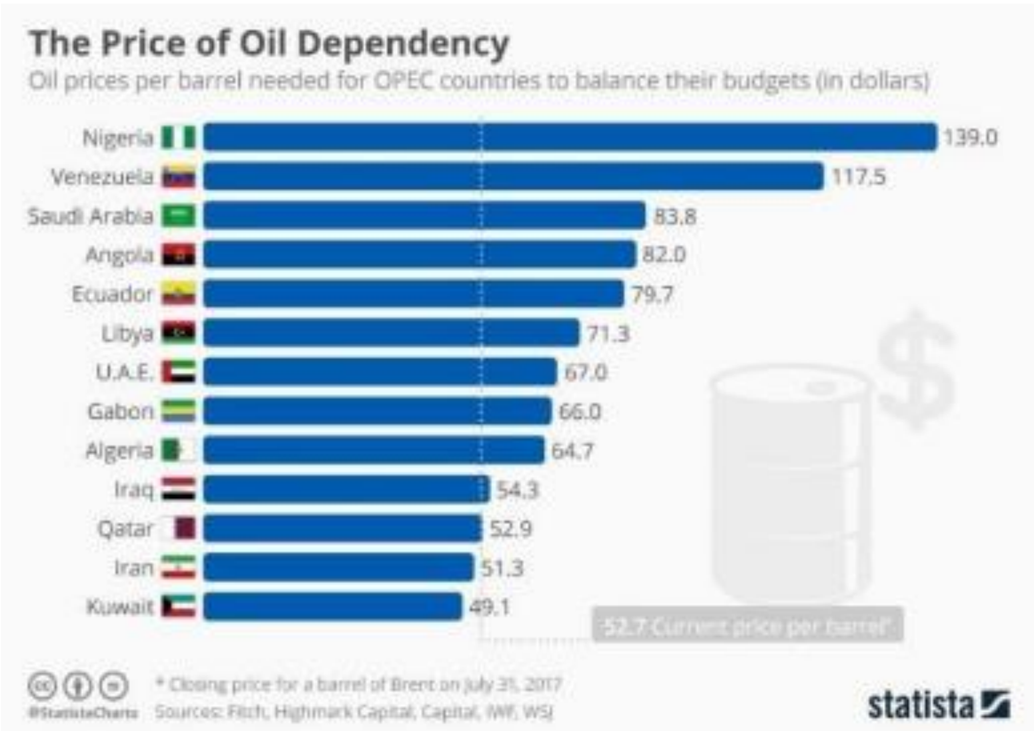
11. What is the recent situation in Venezuela? Watch these 4 videos.

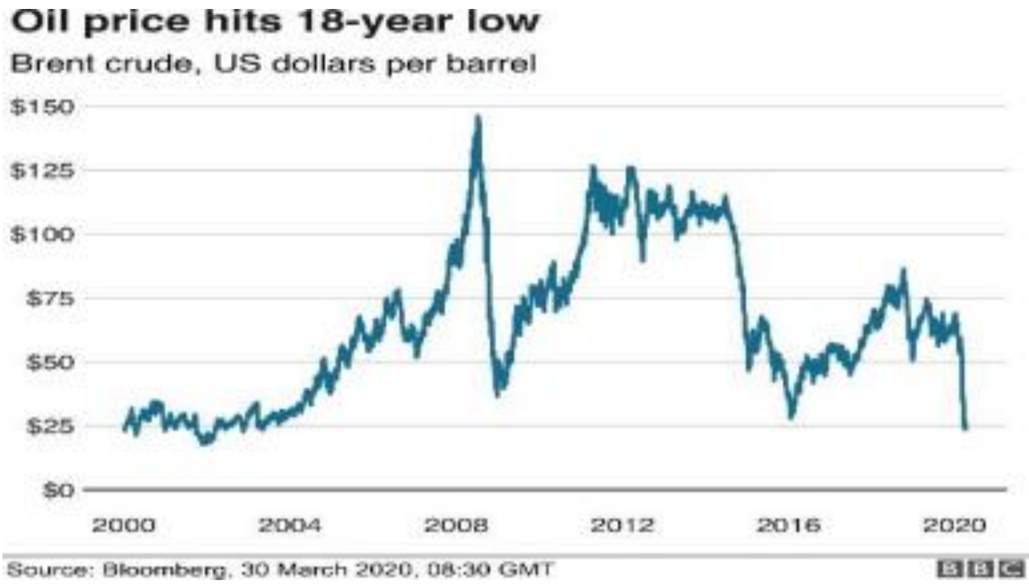
In particular, focus on the impact of inflation. What does inflation mean? inflation definition What impact does it have on day to day living and working in Venezuela? https://www.tutor2u.net/economics/reference/economic-crisis-in-venezuela What is inflation? https://www.tutor2u.net/economics/reference/why-is-high-inflation-a-problem

[The story of Venezuela's collapse]

Finding out more/further investigation

1. Look at the 2 charts below. What impact might the current oil price of $25 per barrel have on the Venezuelan economy?





Task 2

Resource Curses.

A key (economic) question is why some countries are richer than others. Watch the videos about natural resource curses which offer a potential explanation

Is there a natural resource curse?

Innovating your way out of the resource curse An interesting article about the impact of natural resources on Qatar and Norway and policies to diversify away from sole reliance on a primary product.

Micro economics tasks

Task 1 Choconomics

As an introduction to the subject, I am asking you to focus on the economics of chocolate.

You will watch the short Fairtrade Foundation video about cocoa production and answer a range of questions as you go along.

Some of the questions are **extension activities (with additional links)** should you wish to investigate the topic further.

Remember, you are only just beginning your economics journey and I am not expecting you to understand everything. My aim is to introduce you to a wide range of typical ideas, theories and examples that we might study throughout the course. Fairtrade Worksheet: week 1 bridging

Watch [The story of chocolate : unwrapping the bar]and use it to help you answer the questions below. You do not need to print the worksheet out. You can type your answers directly onto the worksheet or write out by hand. I suggest reading through all the questions first. You may have to watch the video more than once.

1. Name two of the world's largest producers of cocoa.

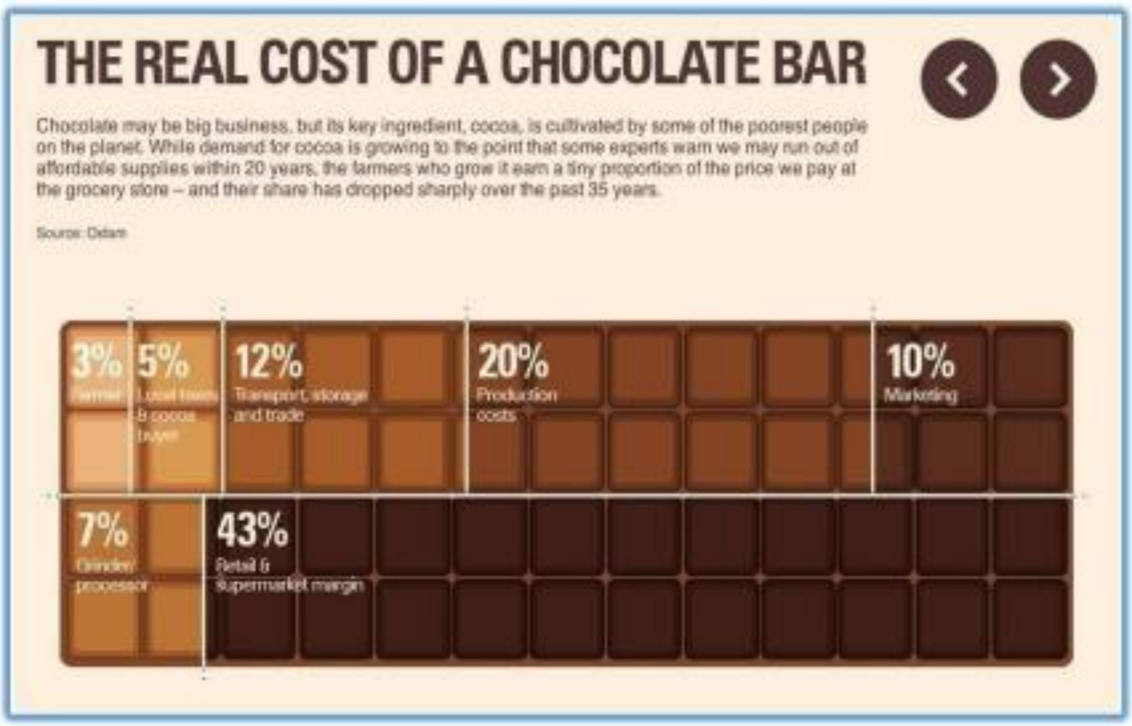
2. How long does it take for a cocoa pod to grow?

3. Watch the harvesting of the cocoa pod and explain what is meant by specialisation and division of labour.

4. Explain what is meant by a labour intensive industry? With reference to cocoa, how does this impact the farmers' ability to grow more (expand supply)? How might this impact a farmer's standard of living?

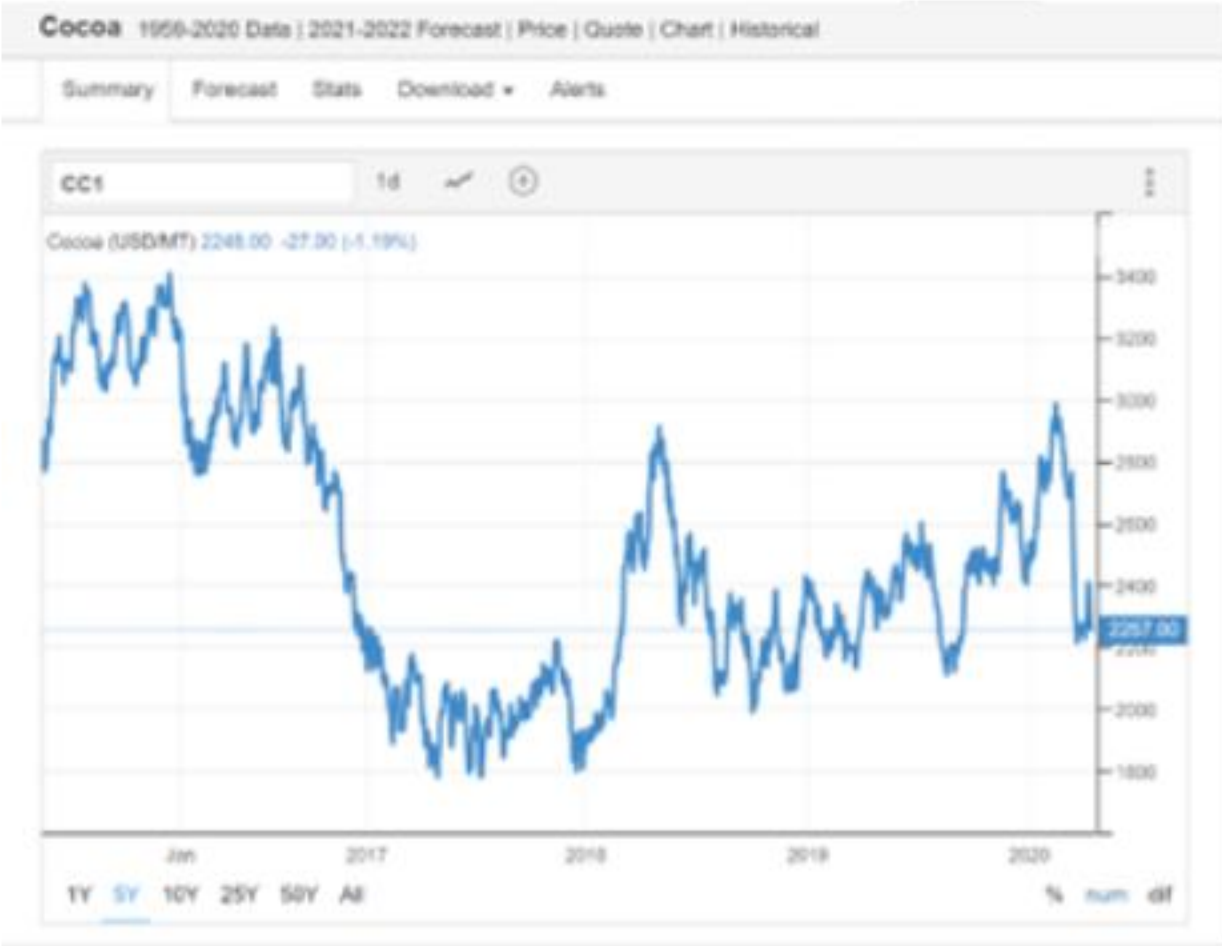
5. What external threats are there to the cocoa harvest? Eg drought. What impact would this have on a cocoa farmer's annual earnings?

6.



Extension question:

Look at the 'real cost of a chocolate bar' graphic. Using your own knowledge, explain why the farmer receives such a small percentage of the sales price (and thereby lives under the poverty line)?



7. **Extension question:** Look at the world cocoa price graph and describe what has happened to the price over the past 5 years.

8. **Extension (Independent research) question**

Explain 3 factors that might influence the world cocoa price.

eg a rise in price in 2018 from 1800 USD/MT to 2900 USD/MT might be due to a crop disease that kills 80% of the Ivory Coast's cocoa pods. This leads to a shortage of cocoa and the market price will rise

9. Explain what is meant by a minimum price (for cocoa)? [ignore the diagrams]. How will this help the Ivory Coast's farmers?

10. Explain 2 benefits to the Ivory Coast of investing the Fairtrade premium in building nursery schools and installing water pumps? (consider the wider economic benefits e.g. to workers/parents/the government?)

11. **Extension question** :
Read the study note about Opportunity cost and go back to your answer to question 10. Can you apply the concept of opportunity cost to the collection of water? Prior to the installation of the water pump, where/how might the villagers have accessed water? What choices might a villager have to make when collecting water previously? What activities did they have to give up?

12. Why is important for the (women) farmers to diversify? What crops have they diversified into?

13. **Extension: dangers of over specialisation**
Explain why such heavy dependency on 1 industry is a risk for a country such as the Ivory Coast dangers of overspecialisation

14. Why does the farmer say he might have to move into producing palm oil?

15. **Extension question : objectives of producers/functions of the price mechanism**
Read/watch the explanations about the objectives of producers / functions of the price mechanism. Go back to question 14, explain why the farmer is behaving rationally when considering a move into palm oil?

16. Overall conclusion: Evaluate whether you think the Fairtrade scheme in the Ivory Coast is successful. You should give a balanced answer giving 1 reason for and 1 reason against before coming to an overall judgement.

Year 11 into Year 12
English Language & Literature

Welcome to A Level English Language and Literature!

Hopefully you've chosen to study English Language and Literature because:

- You're interested in different people in society and their experiences
- You like stories
- You're interested in the choices that writers make and the impact that they have on readers
- You like non- fiction writing as well as fiction
- You like discussion

Set texts:
The Handmaid's Tale by Margaret Atwood
Mean Time poetry anthology by Carol Ann Duffy
The Great Gatsby by F. Scott Fitzgerald
A Streetcar Named Desire by Tennessee Williams

You could also watch any TV/ film adaptation of these set texts and read any other work by the authors.

General
Bill Bryson, Mother Tongue
Victoria Fromkin, Robert Rodman and Nina Hyams, An Introduction to Language
Sara Thorne, Mastering Advanced English Language
R L Trask, Language the Basics
Crystal, A Little Book of Language

Language and mode
Ursula Clark, An Introduction to Stylistics
David Crystal, Rediscover Grammar (for reference)

USEFUL WEBSITES
http://www.universalteacher.org.uk/
Lots of invaluable information on all aspects of your English Language and Literature course.

www.englishlangsfx.blogspot.com/
Written by Dan Clayton at St Francis Xavier College; provides excellent materials and discussions, plus lots of links to other relevant sites.

http://www.ling.lancs.ac.uk/
Lancaster Uni Language and Linguistics site. Enormous. They have a specific A Level site within this and loads of resources.

http://www.geoffbarton.co.uk/ useful site which covers many aspects of A Level.

http://www.phon.ucl.ac.uk/ Very good for dialects, phonology.

Activities that you could be doing to get a head start on the course:

- Read the set texts
- Make personal notes on plot, character, theme and key moments in the texts
- Read articles about the texts and any interviews that you can find with the authors too (keep track of these because they will be useful when the course starts)
- Read as widely as possible around the Canon of English Literature- lots of good starting points here https://www.goodreads.com/shelf/show/literary-canon

Great Works of Literature: Get Reading!

https://www.planetebook.com/free-ebooks/1984.pdf Read it here: 1984 – George Orwell

https://epdf.pub/atonementa44b158966bc8442b357d3883d5593d543983.html Read it here: Atonement – Ian McEwan

https://www.tyrone.k12.pa.us/site/handlers/filedownload.ashx?moduleinstanceid=2789&dataid=3197&FileName=On%20the%20Road.pdf Read it here: On The Road – Jack Kerouac

https://www.fadedpage.com/books/20171024/html.php Read it here: As I Lay Dying – William Faulkner

https://docs.google.com/viewer?a=v&pid=sites&srcid=YW5udXJpc2xhbWljc2Nob29sLm9yZ3xzaXN0ZXIta2F0ZWx5bnxneDo2NjVmZmE1NzNjNjc4NWM Read it here: To Kill A Mockingbird – Harper Lee

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http://mrsfieldstchs.weebly.com/uploads/3/7/7/1/37719247/the_road_-_text.pdf Read it here: The Road – Cormac McCarthy

https://file.largepdf.com/file/2019/08/28/The%20Handmaid_s%20Tale.pdf Read it here: The Handmaid's Tale – Margaret Atwood

http://online.fliphtml5.com/kmfb/xrnv/#p=4 Read it here: Kite Runner – Khaled Hosseini

https://www.chinhnghia.com/1836.pdf Read it here: In Cold Blood – Truman Capote

https://www.uzickagimnazija.edu.rs/files/Catcher%20in%20the%20Rye.pdf Read it here: Catcher in the Rye – JD Salinger

https://d2ct263enury6r.cloudfront.net/X2bpH13Xnjn4ZJspWQzb5LMu7BGp5CUGaPGFQqVXvLT2M1AW.pdf Read it here: Lord of the Flies – William Golding

https://archive.org/details/in.ernet.dli.2015.461023/page/n1/mode/2up Read it here: Rebecca – Daphne du Maurier

https://www.matermiddlehigh.org/ourpages/auto/2012/11/16/50246772/Beloved.pdf Read it here: Beloved – Toni Morrison

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https://www.yumpu.com/en/document/read/7849757/never-let-me-go-kazuo-ishiguro Read it here: Never Let Me Go – Kazuo Ishiguro

English Literature

Welcome to A Level English Literature!

Hopefully you've chosen to study English Literature because:

- You're interested in different time periods and the literature created in those times
- You're interested in different literary thoughts and concepts
- You want to learn about writers' lives and their influences
- You like discussing language and meaning
- You're good at discussion and debate
- You're interested in history

Set texts: Paper 1

Othello by William Shakespeare
The Great Gatsby - F. Scott Fitzgerald
Love through the ages

You could also watch any TV/ film adaptation of these set texts and read any other work by the authors.

USEFUL WEBSITES

https://www.novelguide.com/- Plot summaries etc.

https://www.ipl.org/div/litcrit/- Literary criticism (reading around the text/ author)

https://www.teachitenglish.co.uk/ks5-lit

https://poetryarchive.org/poetryarchive/home.do

http://www.literaryhistory.com/

https://www.litencyc.com/

https://www.bartleby.com/cambridge/index.html

https://blackwell-compass.com/subject/literature/

Activities that you should be doing to get a head start on the course:

- Read the set texts
- As you read make personal notes on plot, character, theme and key moments in the texts. This can be shaped as chapter summaries, scene summaries, character maps
- Read articles about the texts and any interviews that you can find with the authors too (keep track of these because they will be useful when the course starts)
- Research the following authors:
    - William Blake
    - Robert Burns
    - Thomas Hardy
- Read as widely as possible around the Canon of English Literature- lots of good starting points here https://www.goodreads.com/shelf/show/literary-canon
  I suggest also researching/reading texts by the following authors where they appear on paper 2:
- §Tennessee Williams
- §Margaret Atwood
- §Carol Ann Duffy
- §Owen Sheers
- §Jeanette Winterson
- §Arthur Miller

Great Works of Literature: Get Reading!

https://www.planetebook.com/free-ebooks/1984.pdf Read it here: 1984 – George Orwell

https://epdf.pub/atonementa44b158966bc8442b357d3883d5593d543983.html Read it here: Atonement – Ian McEwan

https://www.tyrone.k12.pa.us/site/handlers/filedownload.ashx?moduleinstanceid=2789&dataid=3197&FileName=On%20the%20Road.pdf Read it here: On The Road – Jack Kerouac

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https://www.chinhnghia.com/1836.pdf Read it here: In Cold Blood – Truman Capote

https://www.uzickagimnazija.edu.rs/files/Catcher%20in%20the%20Rye.pdf Read it here: Catcher in the Rye – JD Salinger

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https://www.yumpu.com/en/document/read/7849757/never-let-me-go-kazuo-ishiguro Read it here: Never Let Me Go – Kazuo Ishiguro

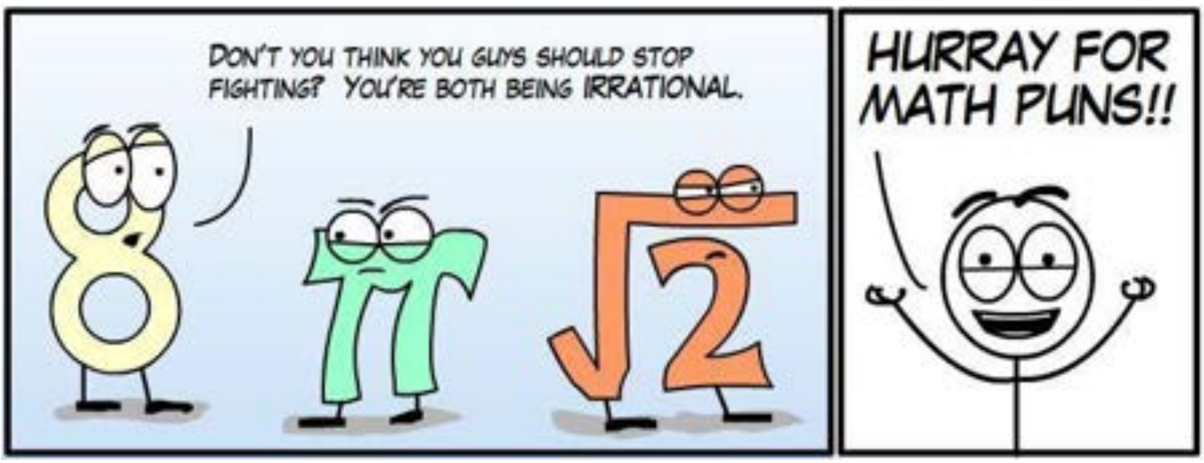
FURTHER MATHEMATICS

INDUCTION TASK

SUMMER HOLIDAY WORK

Please work through the booklet answering any questions that have been set. Research anything you do not know. In class you will peer mark the exercises. At the end there is an assessment sheet. Please answer these questions on paper with clear workings. This will be handed into your maths teacher for marking at the beginning of September along with an A4 poster explaining different number systems.

ENJOY!!



Different Number Systems

Research the following Number Systems. Write down a sentence describing each system and then write down the symbol used for each one.

Integers

Natural Numbers

Rational Numbers

Irrational Numbers

Real Numbers

Complex Numbers

Number Systems

Write down two examples of each of the following

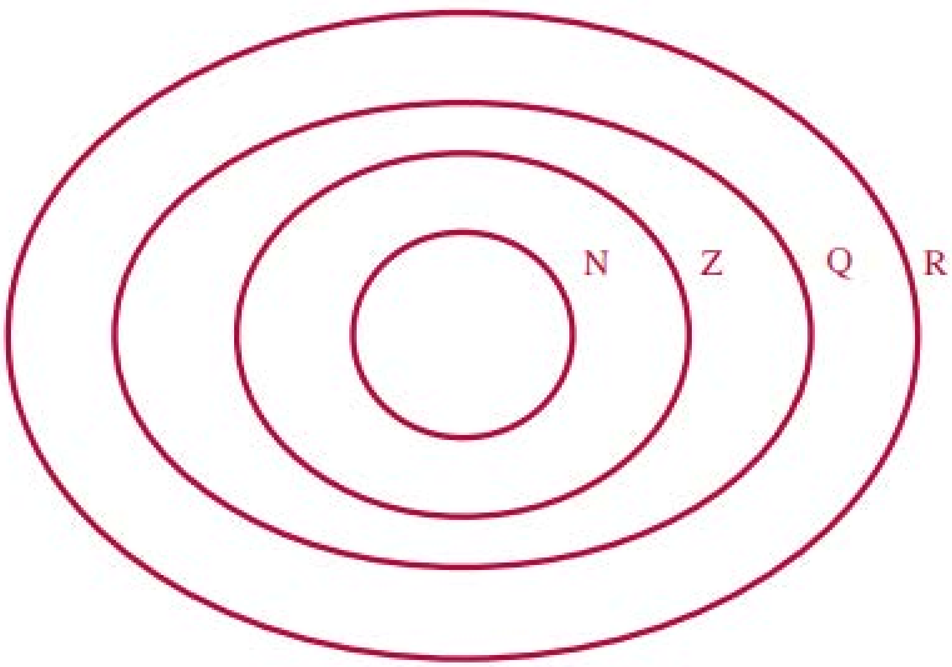
Natural numbers	
Positive Integers	
Negative Integers	
Integers	
Rational numbers which are positive and not natural numbers	
Rational numbers which are also integers	
Irrational numbers	
Real numbers	
Real numbers which are natural numbers, integers and rational numbers	
Complex numbers	

More Number Systems

Place the following numbers in their correct positions in the Venn Diagram below.

$5, \quad \sqrt{5}, \quad \frac{15}{3}, \quad -\frac{2}{3}, \quad 0.3333333 \ldots \ldots,$

$0.27272727 \ldots \ldots \ldots., \quad \sqrt{4}, \quad \pi, \quad -3$



Complex Numbers

Remember $i = \sqrt{-1}$ $i^2 = -1$

Using these facts

Simplify

1. i^5

2. i^9

3. i^{33}

4. $i^{16} + i^{10} + i^8 - i^{14}$

5. $i^{12} \times 3i^2 \times 2i^8$

6. $4i^3 + 7i^9$

7. $13i^8 - 4i^{14}$

8. $(3i^5)^2$

Fill in the real and imaginary parts of each complex number

Complex Number	Real part	Imaginary part
3		
0		
$2 + 7i$		
$4 - 3i$		
$-5 + 7i$		
$\frac{2}{3} + 5i$		
$2i$		
i		
$7 - \frac{4}{11}i$		
$5 + 6i$		
9		
$-\frac{1}{2} - \frac{2}{3}i$		

Adding and Subtracting Complex Numbers

To do this collect the real parts together and then collect the imaginary parts.

e.g. $(2 + 3i) + (4 - i) = (2 + 4) + (3 - 1)i = 6 + 2i$

$(7 - 4i) - (5 -2i) = (7 - 5) + (-4 + 2)i = 2 - 2i$

Now you do:

1	$(12 + 4i) +(7 - 11i)$	
2	$(7 - 2i) + (9 - 4i)$	
3	$(4 - 6i) + (-5 - i)$	
4	$(3 - 8i) - (2 - 4i)$	
5	$(-12 - 5i) - (-2 - 8i)$	
6	$\left(2+\frac{1}{3}i\right)+\left(3-\frac{5}{6}i\right)$	
7	$\left(4+\sqrt{-16}\right)+\left(-5-\sqrt{-25}\right)$	
8	$z_1= 5 + I$, $z_2= -4 + 6i$ $z_3=-11 + 2i$ Calculate $(z_1+z_2)-z_3$	
9	$\left(4-\sqrt{-50}\right)-\left(3+\sqrt{-8}\right)$	

10	$z_1 = a + bi$ $z_2 = c + di$ $z_1 + z_2 =$ $z_2 + z_1 =$ $z_1 - z_2 =$ $z_2 - z_1 =$	

Square roots of negative numbers

We can use $\sqrt{ab} = \sqrt{a}\sqrt{b}$ to write square roots of negative numbers in terms of i.

e.g $\sqrt{-36} = \sqrt{36}\sqrt{-1} = 6i$

$\sqrt{-100} = \sqrt{100}\sqrt{-1} = 10i$

Now you write the following in terms of i as simply as possible

1. $\sqrt{-49}$

2. $\sqrt{-81}$

3. $\sqrt{-1.21}$

4. $\sqrt{-0.04}$

5. $\sqrt{-8}$

Solving quadratic equations

Complex numbers can be used to solve quadratic equations when the number being square rooted in the quadratic formula is negative.

Now use the quadratic formula to solve the following quadratic equations.

1. $x^2 + 6x + 13 = 0$

2. $x^2 - 4x + 13 = 0$

3. $2x^2 - 2x + 5 = 0$

4. $x^2 - 10x + 34 = 0$

Into the Sixth Form - Further Maths Initial Complex numbers Assessment

Answer the following questions on paper showing clear workings. This needs to be handed into your teacher for marking.

1. Write the following in terms of bi where $b \in$ R

a) $\sqrt{-36}$ b) $\sqrt{-121}$ c) $\sqrt{-12}$ d) $\sqrt{-300}$

2. Simplify the following, giving your answer in the form a + bi, where $a \in$ R and $b \in$ R.

a) (3 + 6i) + (7 + 3i) b) (4 — 5i) + (— 4 + 7i)

c) (2 + 9i) — (7 + i) d) (3 — 4i) — (6 — 10i)

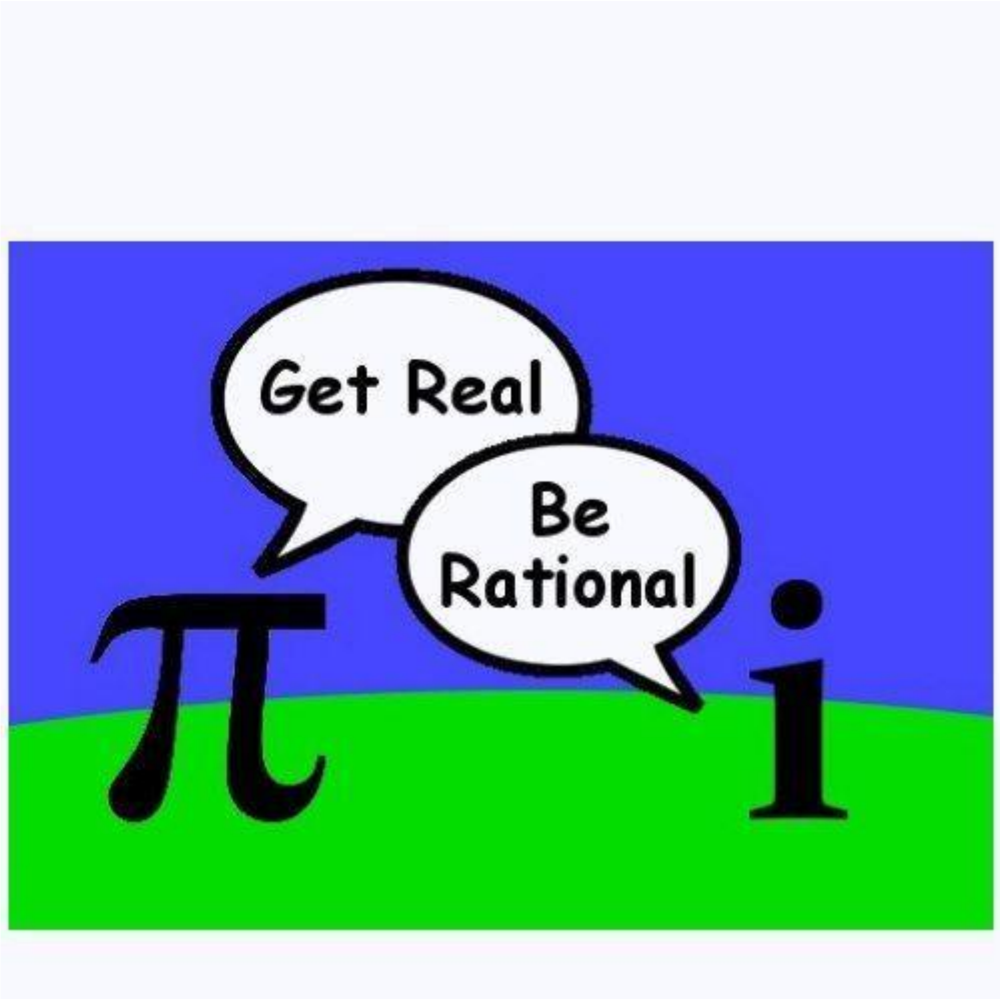
e) (3 + 4i) – (3 — 4i) f) $\left(\frac{1}{2}+\frac{1}{3}i\right)+\left(\frac{5}{2}+\frac{5}{3}i\right)$

3. Solve these equations

a) $x^2+4x+29=0$ b) $x^2-6x+18=0$

c) $x^2+5x+25=0$ d) $x^2+3x+5=0$

The final piece of your assessment is to design a poster on an A4 piece of paper explaining and describing the different number systems. The poster is optional but if you do make one, please bring it in to the lesson to show your teacher.


Get Real
Be Rational

Future A-Level Geographers

Hello Year 11 Geographers,

If you are considering Geography A-Level next year or are just interested in continuing to develop your geography knowledge over the coming weeks, why not use this time to explore the subject a little more. These pages are just ideas that you can dip in and out of as much as you like or use as a starting point to find other resources you find interesting – Enjoy!

Mrs Gent,, Mr Forster-Rea and Miss Biggin

Follow the Geography Department Twitter for more geography updates, recommended articles and interesting facts!

@Campiongeog

OPEN LEARN

The Open University have a huge number of free online courses that you can study. They tell you how long they take and give an outline of what is included. You also get a free statement of participation on completion. Take a look and see if any take your interest. Some recommended ones linked to the A Level course are listed below:

https://www.open.edu/openlearn/nature-environment/the-environment/environmental-science/managing-coastal-environments/content-section-0?active-tab=description-tab

https://www.open.edu/openlearn/science-maths-technology/science/environmental-science/global-water-resources/content-section-0?active-tab=description-tab

WHAT TO WATCH?

THE ECONOMIST VIDEOS

https://www.youtube.com/user/EconomistMagazine/videos

E.g. 'Mangroves: How they help the ocean?' and 'Can you insure against climate change?'

TED TALKS

https://www.ted.com/talks?sort=newest&topics%5B%5D=Global+issues

SIR DAVID ATTENBOROUGH BOX SETS

https://www.bbc.co.uk/iplayer/search?q=david+atten

UNDRR STOP DISASTER EMERGENCY PLANNING GAME

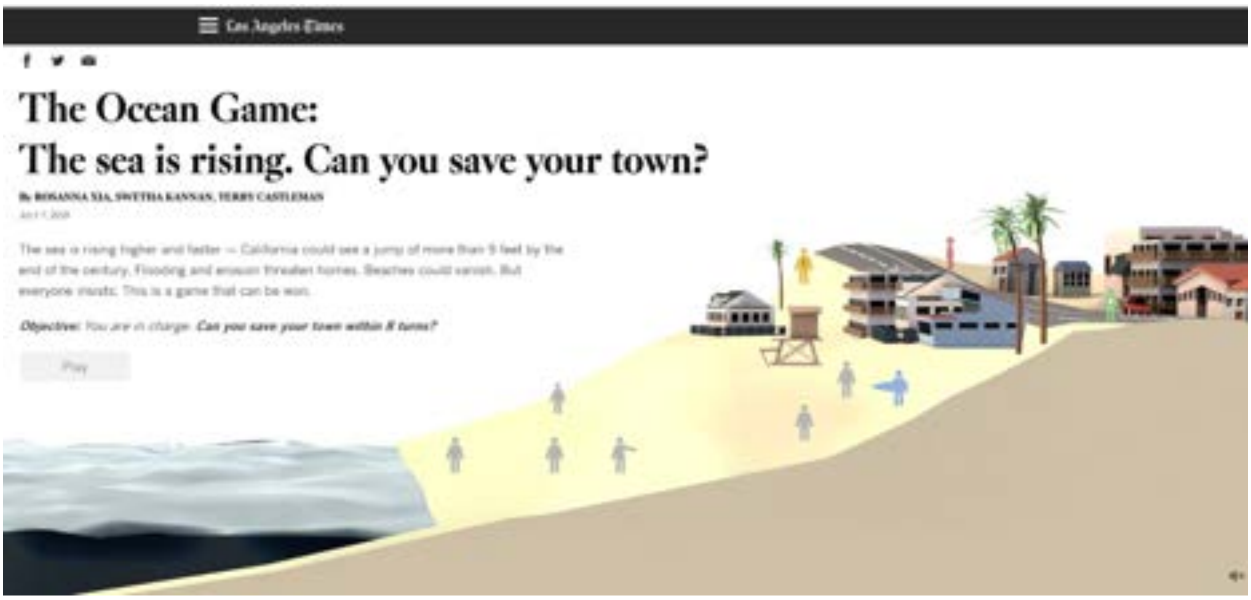
https://www.stopdisastersgame.org/stop_disasters/

UNDRR stands for the UN Office for Disaster Risk Reduction. This online game aims to teach you how to build safer villages and cities against disasters. You will learn how the location and the construction materials of houses can make a difference when disasters strike and how early warning systems, evacuation plans, and education can save lives. You are the future architects, mayors, doctors, and parents of the world of tomorrow. If you know what to do to reduce the impact of disasters, they will create a safer world.

LA TIMES CLIMATE CHANGE RISING OCEANS GAME

https://www.latimes.com/projects/la-me-climate-change-ocean-game/

The sea is rising higher and faster — California could see a jump of more than 9 feet by the end of the century. Flooding and erosion threaten homes. Beaches could vanish. But everyone insists: This is a game that can be won.



WHAT TO LISTEN TO?

Search for these Podcasts on your phone!

THE ECONOMIST PODCASTS

https://www.economist.com/podcasts/

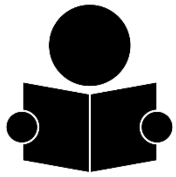
ASK THE GEOGRAPHER – Royal Geographical Society

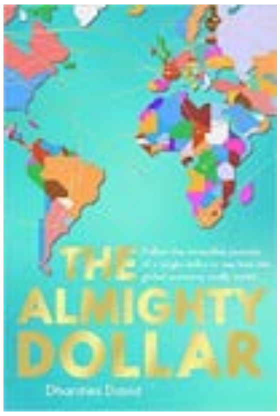
https://www.rgs.org/schools/teaching-resources/ask-the-expert-podcasts/

You can then listen on Soundcloud or iTunes

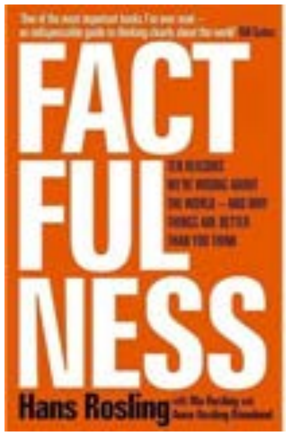
'Hazards and volcanic gas emissions' with Dr Tom Pering, 'Forests and the Carbon Cycle' with Professor Rob MacKenzie and Professor Jerry Pritchard, 'Extreme weather in the UK; past, present and future' with Professor Georgina Endfield.

WHAT TO READ?

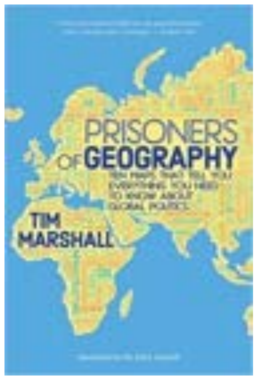




Darshini David - The Almighty Dollar



Hans Rosling – Factfulness



Tim Marshall – Prisoners of Geography

EXPLORE CAREERS IN GEOGRAPHY

Explore the links below to see the further education and career opportunities linked to Geography!

Royal Geographical Society with IBG

Click the link below and at the bottom of the page click on the '***Going Places with Geography Brochure'*** link. Explore why Geography is so valuable to study, the skills you gain through your studies and how these can link to future careers.

https://www.rgs.org/schools/teaching-resources/going-places-with-geography-brochure/

The link below takes you to a page where you can see the huge range of jobs geographers can go onto do!

https://www.rgs.org/iamageographer/

HOW TO IMPROVE YOUR GENERAL GEOGRAPHY KNOWLEDGE ...

Use Sporcle to see how good your country and capital city knowledge is.

Start with Europe ...

https://www.sporcle.com/games/g/europe

Transition materials for A Level History

Topic: Germany 1919-1989

Introduction

Welcome to A-Level History! Firstly, congratulations on completing your GCSE's and well done for choosing History! History is a challenging A-Level which involves lots of content and so you will need to take some time to read and prepare.

In this pack you will find plenty of materials to prepare you for studying the Germany section of your course. It is aimed for you to use once you have completed your GCSE's before starting you're A-Levels in September. Consider this your summer work to get you ready and up to scratch! A-Levels are tricky qualifications to achieve and you need to ensure that you are fully prepared for them. The more work you do now, the easier you will find it when you start your course.

Key things to do before starting the course:

- Get yourself a folder and dividers – there is a lot to learn so it is best to be organised from the very start.
- Ensure that you know what exam board you are studying. Germany is a topic that appears on many exam boards but it is always helpful to know what your specific board requires. You can download this from the exam board website.
- Read this booklet carefully and work through the activities as instructed by your teacher. This will give you the foundation knowledge needed to start your course.

Contents list

1. **Reading list**

These are some key things that you should look at in order to prepare for your Germany module.

2. Independent Research

A-Levels are independent course and you will be expected to study independently as well as undertake your lessons. These tasks will give you a starting point in what and how to research independently.

3. Required Knowledge and Skills

You will have to have some basic knowledge and historical skills before undertaking your A-Level. There are 10 activities for you to complete which will prepare you for this.

4. A Base line Assessment

This will test the knowledge and skills that you have already before starting your course. It is a good idea to do this last, after having completed the set research and tasks in this booklet.

5. The world is out there

History is not just in books or the classroom. It is always a great idea to take some time to visit places to enrich your historical experience. This will provide you with some ideas on how to enrich your learning.

1. <u>Reading List</u>

Recommended Non Fiction:

Lynn Abrams, Bismarck and the German Empire 1871-1918, Lancaster Pamphlets, (2006)

Alan Bullock, Hitler A Study in Turanny, Penguin, (1990)

David Evans and Jane Jenkins, Years of Weimar and the Third Reich, Hodder and Stoughton, (2004)

Mary Fulbrook, Divided Nation; a History of Germany, OUP, (1992)

Fulbrook, Williamson, Fellows and Wells, Democracy and Dictatorship in Germany 1919-1963, Heinemann, (2008)

Dick Geary, Hitler and Nazism, Routledge (2000)

William Glenn Grey, Germany's Cold War: The Global Campaign to Isolate East Germany 1949-1969, University of North Carolina Press, (2003)

Ian Kershaw, Making Friends with Hitler, Penguin, (2005)

Martin Kitchen, A History of Modern Germany 1800-present, Wiley-Blackwell (2012)

A Kitson, Germany 1858-1991, Hope Terror and Revival, Oxford Advanced History, (2001)

Geoff Layton, Germany: The Third Reich 1933-1945, Hodder Murray, (2010)

Geoff Layton, Democracy and Dictatorship in Germay 1919-1963, Hodder, (2015)

S Lee, Hitler and Nazi, Routledge (1998)

D Murphy (ed.) Germany 1870-1991, Collins, Flagship History (2000)

William L Shirer, The rise and fall of the Third Reich, Mandarin, (1991)

Recommended Films

The Wonderful and Horrible Life of Leni Riefenstahl. (1993) – A documentary of the film director's life.

Cabaret (1972) – Oscar winning musical set in Weimar Germany

The Harmonists (1997) – A musical group's story in Weimar Germany

Hitler, the Rise of Evil (2003) – Dramatized story of Hitler's career

Triumph of the Will (1935) – Propaganda film on Hitler

The Lives of Others (2006) – Life in East Germany during the Cold War.

The Pianist (2002) – Life of Jewish musician during Nazi reign

Schindler's list (1996) – showing how people attempted to escape the Holocaust

Anne Frank The Whole Story (2006) – two part mini-series based on the Diary of Anne Frank.

2. <u>Independent Research</u>

Independent research is a key part of studying at A-Levels. Below are some research activities for you to undertake in order to prepare you for starting your History A-Level unit in German history.

Research Activity 1: Research the views of the following historians on German History. Find out about their life and work. Summarise their views and why they have been influential in the history of Germany.

Ian Kershaw

https://www.amazon.co.uk/Hitler-Ian-Kershaw/dp/0141035889?ie=UTF8&*Version*=1&*entries*=0

http://news.bbc.co.uk/1/hi/uk/2045979.stm

Richard Overy

https://www.amazon.co.uk/Dictators-Hitlers-Germany-Stalins-Russia/dp/0140281495/ref=sr_1_12?s=books&ie=UTF8&qid=1461159812&sr=1-12

https://www.amazon.co.uk/Third-Reich-Chronicle-Richard-Overy/dp/085738175X/ref=sr_1_16?s=books&ie=UTF8&qid=1461159812&sr=1-16

http://www.historytoday.com/author/richard-overy

Research Activity 2: Search for 4 Nazi propaganda posters. Annotate them in as much detail as you can and explain how they would have won support for the Nazi party.

http://www.bc.edu/bc_org/avp/cas/his/CoreArt/prop/propmain.html

Research Activity 3: Create a list of the key stages that led to Germany being divided after the Second World War.

http://www.johndclare.net/cold_war1_Germany.htm

Research Activity 4: Create a table showing the similarities and differences between East and West Germany after 1945.

http://understandinggermany.de/history/differences-between-east-west-germany/

http://www.theguardian.com/world/2015/oct/02/german-reunification-25-years-on-how-different-are-east-and-west-really

http://www.historylearningsite.co.uk/modern-world-history-1918-to-1980/the-cold-war/the-berlin-wall/

Research Activity 5: Listen to and/or read J.F.Kennedy's speech on the Berlin wall in 1963.

http://millercenter.org/president/speeches/speech-3376

https://www.youtube.com/watch?v=0GKd50lrROc

What can we learn about America's view from this speech?

3. Required Knowledge and Skills

Before you start studying Germany for your History A-Level, you need to build up some background knowledge and skills so that you can start the course in the strongest position. Complete the following tasks to prepare you:

Knowledge Tasks

Task 1: Create a detailed timeline of German history from 1871 to 1995. Be sure to include how the country was ruled, who ruled the country, revolutions and wars.

Task 2: Create a fact file on each of Germany's Kaisers between 1871–1918. You must include details of their rule and the impact

- Wilhelm I (1871–1888)
- Friedrich III (9 March-15 June 1888), who ruled for 99 days
- Wilhelm II (1888–1918)

Task 3: Study Germany during the First World War and write an account of their involvement. Be sure to answer the questions: What role did they play on the war? What key battles were they involved in? What impact did the war have on Germany as a country?

Task 4: Create a 'family tree' of the Nazi party. Include all the key individuals and their roles.

TASK 5: Create a glossary of key terms that you will need to know. Use the sheet below to add the definitions to the key terms listed. Add any of your own that you discover during your research.

Glossary

Affidavit	Concordat
Anti feminist Ideology	Council of Europe
Anti Semitism	Cult of Personality
Aryan	DAF
Asocials	Democratic Centralism
Auxiliaries	De-Stalinisation
Basic Law	Dualism
Berlin Wall	ECSC
Bezirke	EEC
Bizonia	Einsatzgruppen
Black Market	Elites
Bormann	Enabling Act
Boycott	Euraton
Bundesrat	Euthanasia
Bundestag	Final Solution
Cabinet	First Past the Post
Capitalism	Five Year Plans
Chancellor	Four Year Plans
Chancellor Democracy	FRG
Coalition	Fuhrerprinzip
Collectivisation	Gastarbeiter
Gauleiters	

GDR	New Course
Genocide	New Order
Gestapo	Oder-Neisse Line
Ghettos	OEEC
Gleischaltung	Parliamentary Council
Hallstein Doctrine	Patriotism
Herrenvolk	People's Community
Hitler Youth	Plebiscite
Ideology	Politburo
Indoctrinate	President
Judiciary	Propaganda
Junkers	Proportional Representation
Kripo	Putsch
Knristallnacht	Racial Genetics
Labour Exchanges	Reichstag
Lander	RSHA
Landerkammer	Ruhr
Law 131	SA
Lebensraum	Saar
Marshall Plan	SD
NATO	Waffen SS
Seven Year Plan	Wannsee

Social Darwinism Social Justice Spiessburger SS Stalinist Stasi Teutonic Paganism Totalitarian Treaty of Rome Treaty of Versailles Trizonia Truman Doctrine Untermenschen Volk Volkisch Volkseigene Betriebe Volksgemeinschaft Volksammer Weimar Welfare State	Any words of your own? Write them below!

Skills Tasks

Task 6: *Interpretations.* What do you think about the historian's opinion below? Assess the strengths and weaknesses of the interpretation.

'It was always inevitable that Hitler would come to power.'

Task 7: *Forming your opinion.* Part of history is all about having your own opinions and arguments. Do you think it was inevitable that Hitler would come to power in Germany?

Task 8: *Source Analysis.* Look at the Nazi 25 point programme. What can we learn from this source? How useful is this as evidence to historians studying Germany?

The Nazi Party Programme

We demand:

The unity of all German-speaking peoples.

The abolition of the Treaty of Versailles.

Land and colonies to feed Germany's population.

Only Germans can be citizens. No Jew can be a German citizen.

People in Germany who are not citizens must obey special laws for foreigners.

Only German citizens can vote, be employed or hold public office.

Citizens are entitled to a job and a decent standard of living. If this cannot be achieved, foreigners (with no rights as citizens) should be expelled.

No further immigration of non-German must be allowed. All foreigners who have come to Germany since 1914 must be expelled.

11. All payments to unemployed people should end.
12. All profits made by profiteers during the war must be shared.
13. Nationalisation of public industries*.
14. Large companies must share their profits.
15. Pensions must be improved.
16. Help for small shops and businesses; large department stores** must be closed down.
17. Property reform to give small farmers their land.
18. An all-out battle against criminals, profiteers, etc., who must be punished by death.
19. Reform of the law to make it more German.
20. Improve education so that all Germans can get a job.
21. Improve people's health by making a law for people to do sport.
22. Abolition of the Army, and a new People's Army in its place.
23. German newspapers must be free of foreign influence.
24. Freedom of religion.
25. Strong central government with unrestricted authority.

Task 9: *Source Analysis.* Look at the source below. Annotate it in as much detail as you can. What can you learn from this source about the Berlin wall?

"See how many are staying on our side."

Don Wright. *The Miami News*, 1961.

Task 10: *Essay skills.* Look at the question below. How would you structure an answer to this question. Write an essay plan showing your structure.

How important was propaganda in enabling Hitler to maintain control of Germany?

4. Base Line Assessment

This assessment is to give your teacher an understanding of your existing skills and knowledge. Answer the questions as best you can using the research, knowledge tasks and skill tasks that you have been practising throughout this booklet.

1. Assess the impact of the First World War on Germany. (10 marks)

2. Explain the role of a Kaiser and how this affected life in Germany. (10 marks)

3. Using your wider reading and research, discuss the various ways that Hitler has been interpreted in History. What evidence is there that supports or opposes these views? (20 marks)

4. Evaluate the impact the division of Germany had on Germany as a country after the Second World War. (10 marks)

5. The World Is Out There…

There are plenty of opportunities out there to enrich your understanding of German history. The following will be greatly beneficial to your learning and prepare you for your A-Level course.

Places to Visit in Britain

Visit the Holocaust exhibit at the Imperial War Museum, London.

http://www.iwm.org.uk/exhibitions/iwm-london/the-holocaust-exhibition

Visit the The Keep Museum, Dorchester, where they have Hitler's desk.

http://www.keepmilitarymuseum.org/info/the+keep+today

Visit The British Museum, London, where they have an exhibition on Germany as a nation.

http://www.britishmuseum.org/whats_on/past_exhibitions/2014/germany_memories_of_a_nation.aspx

Visit the RAF museum – the London one is just interesting, but the Cosford has a specific Cold War exhibition!

http://www.rafmuseum.org.uk/

Places to Visit Abroad

Visit Auschwitz and get a first-hand experience of the Holocaust.

http://auschwitz.org/en/visiting/

Visit Berlin to see the impact of Germany being split into East and West, the remains of the Berlin wall, and the new Hitler exhibition in the German Historical Museum.

http://www.visitberlin.de/en/spot/deutsches-historisches-museum-german-historical-museum

Produced by Lucy James

Boards studying Germany A-Level are:

OCR: Democracy and Dictatorship 1919-1963

Edexcel: The German Democratic Republic 1949-1990

Germany and West Germany 1918-1989

Germany, 1871–1990: united, divided and reunited

AQA: The quest for political stability: Germany, 1871–1991

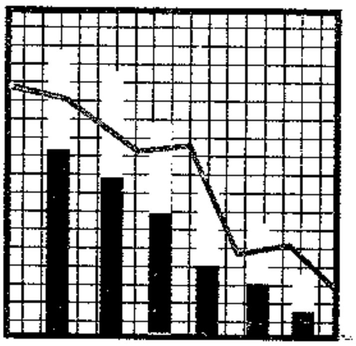
Democracy and Nazism: Germany, 1918–1945

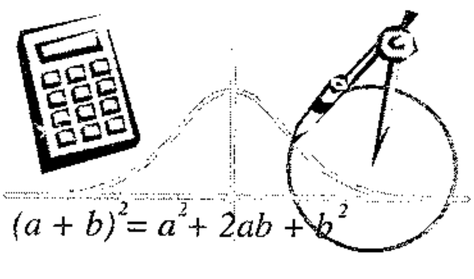
WJEC: DEMOCRACY TO DICTATORSHIP c.1918-1945

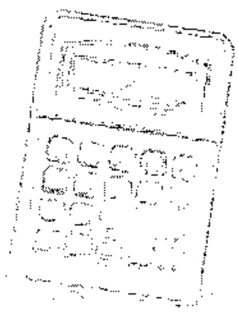
Part 1: WEIMAR AND ITS CHALLENGES c.1918-1933

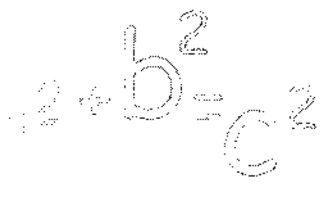
Part 2: NAZI GERMANY c.1933-1945

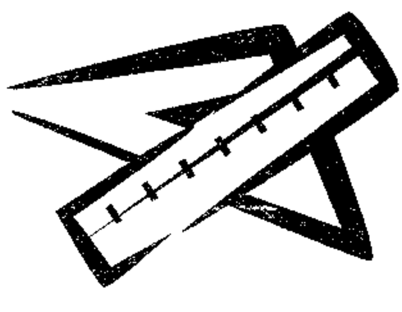
SIXTH FORM MATHS INDUCTION BOOKLET











Contents

Section 1 — Integers, Fractions and Real Numbers

Types of Numbers 1
Fractions 3

Section 2 — Laws of Indices

Laws of Indices 6

Section 3 — Basic Algebra

Factorising 11
Algebraic Fractions 12
Changing the Subject of a Formula 15

Section 4 — Quadratic Equations

Quadratic Equations 17
Factorising $y = ax^2 + bx + c$ 18
Factorising $y = ax^2 + bx + c$ when $a \neq 1$ 20
The Difference of Two Squares 21
Questions on Quadratic Equations 22

Section 5 — Linear Simulataneous Equations

The Elimination Method 23
The Substitution Method 24
Solutions 25

Section 6 — Straight Line Graphs

Finding the Gradient 26
y-intercept, Horizontal and Vertical Lines 28
$y=mx+c$ 29
Finding the Distance Between 2 Points 30

Section 7 — Basic Trigonometry

Basic Trig Ratios 33

Section 8 — Sine and Cosine Rules

The Sine Rule 35
The Cosine Rule 37
Solutions 39

Section 9 — Circles

Circle Properties 40
Cones, Spheres and Circles 41
Questions on Cones, Spheres and Circles 42

Section 10 — Proportion

Proportion 43

Index 44

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Integers

An integer is any positive or negative whole number (including zero).
The set of integers is denoted by the symbol $\mathbb{Z}$ (notice the double line).

Rational, irrational and real numbers

A rational number is any number which can be written as a fraction.
Don't forget, any whole number can be written as a fraction over 1.
The set of rational numbers is denoted by $\mathbb{Q}$.

Recurring decimals are rational.

For example 0.33333...... ($= 0.\dot{3}$) *can be written* $\frac{1}{3}$, *hence it is rational.*

1.6 *is rational because it can be written* $1\frac{6}{10} = \frac{8}{5}$.

A number which cannot be written exactly is irrational.
Irrational numbers are non-repeating decimals, which never end.

The set of real numbers is denoted $\mathbb{R}$. Any rational or irrational number is a real number.

Surds are irrational expressions which contain a $\sqrt{}$ *sign.*
Don't forget $\sqrt{}$ *can also be written* √( ).

The following expressions are in surd form:

$2\sqrt{3}$, $\frac{\sqrt{3}}{2}$, $1 + \sqrt{2}$

$\frac{\sqrt{16}}{3}$ *is not a surd, because* $\frac{\sqrt{16}}{3} = \frac{4}{3}$, *i.e. it is rational.*

Look at these examples:

Are these expressions rational or irrational?

a) $0.\dot{6}$ *b)* 5.26 *c)* $\frac{\sqrt{8}}{\sqrt{2}}$

a) rational because $0.\dot{6} = \frac{2}{3}$

b) rational because $5.26 = \frac{526}{100} = \left(\frac{263}{50}\right)$

c) rational because $\frac{\sqrt{8}}{\sqrt{2}} = \frac{2.828427....}{1.414213....} = 2$

This result could also have been obtained from

$$\frac{\sqrt{8}}{\sqrt{2}} = \frac{\sqrt{4 \times 2}}{\sqrt{2}} = \frac{\sqrt{4} \times \sqrt{2}}{\sqrt{2}} = \sqrt{4} = 2$$

Now you have a go at some:

1) *Are these expressions rational or irrational? Explain your answers.*

a) $\sqrt{2}$ *b)* 0.236 849 *c)* $\sqrt{64}$

d) 2π *e)* $(\sqrt{5})^2$ *f)* $\sqrt{8} \times \sqrt{2}$

2) ***Which of question 1 parts a), c), e) and f) are in surd form?***

3) ***True or false?***
a) All integers are rational.
b) All surds are real numbers.

Solutions

1) a) *irrational.* $\sqrt{2} = 1.414213....$
This is a non-repeating, never ending decimal, so cannot be written as a fraction.

b) *rational.* $0.236\ 849 = \frac{236849}{1000000}$. *Fraction, hence rational.*

c) *rational.* $\sqrt{64} = 8 = \frac{8}{1}$. *Fraction, hence rational.*

d) *irrational.* $2\pi = 2 \times 3.14159....$
$= 6.28318.....$ *Non-repeating, never ending decimal, hence irrational.*

e) *rational.* $(\sqrt{5})^2 = \sqrt{5} \times \sqrt{5} = \sqrt{5\times5}$
$= \sqrt{25}$
$= 5.$

This solution could also have been obtained from $2.23606.... \times 2.23606....$

f) *rational.* $\sqrt{8} \times \sqrt{2} = \sqrt{8\times2}$
$= \sqrt{16} = 4$

2) a) *surd because irrational.*
c) *not surd because* $\sqrt{64} = 8$ *is rational.*
e) *not surd because* $(\sqrt{5})^2 = 5$ *is rational.*
f) *not surd because* $\sqrt{8} \times \sqrt{2} = 4$ *is rational.*

3) a) *True. Any integer can be written as a fraction (e.g.* $\frac{4}{1}$*), so all integers are rational.*
b) *True. Surds are irrational and all irrational numbers are real.*

Rules of fractions

You must be very confident with the four rules of numerical fractions, as the same techniques will be widely used with algebraic fractions.

Adding and subtracting

In order to add or subtract fractions, the denominators must be the same. Hence we look for the lowest common multiple of all denominators (also known as the lowest common denominator).

If you are dealing with mixed numbers, first turn them into top-heavy (or vulgar) fractions.

Look at these examples:

1) $\frac{5}{6} - \frac{3}{4} = \frac{10}{12} - \frac{9}{12}$

$$\boxed{= \frac{1}{12}}$$

2) $3\frac{1}{8} + 1\frac{1}{3} = \frac{25}{8} + \frac{4}{3}$

$= \frac{75}{24} + \frac{32}{24}$ *Don't add the denominators.*

$= \frac{107}{24}$

$$\boxed{= 4\frac{11}{24}}$$

If he can do it, so can you.

Don't forget, multiplying denominators together will always provide a common multiple, although this will not necessarily be the lowest common multiple.

For example, the first example could be solved by:

$$\frac{5}{6} - \frac{3}{4} = \frac{20}{24} - \frac{18}{24} \quad \text{etc.}$$

The technique of multiplying the denominators together is used when adding or subtracting algebraic fractions.

Multiplying

1) Turn mixed numbers into top-heavy fractions.
2) Cancel any numerator and any denominator.
   (This will ensure that the answer is in its simplest form).
3) Multiply the numerators and
4) Multiply the denominators.

Dividing

1) Turn any mixed numbers into top-heavy fractions.
2) Turn the <u>second</u> fraction upside down and change the sign to <u>multiply</u>.
3) Proceed exactly as for multiplying.

Look at these examples:

1) $$\frac{3}{4} \div \frac{3}{16} = \frac{\cancel{3}^1}{\cancel{4}_1} \times \frac{\cancel{16}^4}{\cancel{3}_1} = \frac{4}{1} \quad \boxed{= 4}$$

2) $$2\frac{3}{4} \times 1\frac{2}{3} = \frac{11}{4} \times \frac{5}{3} = \frac{55}{12} \quad \boxed{= 4\frac{7}{12}}$$

3) $$3\frac{1}{2} \div \frac{2}{3} = \frac{7}{2} \div \frac{2}{3} = \frac{7}{2} \times \frac{3}{2} = \frac{21}{4} \quad \boxed{= 5\frac{1}{4}}$$

Now you have a go at some:

1) $\frac{3}{4} + \frac{1}{3}$ 2) $1\frac{5}{6} + 2\frac{1}{2}$ 3) $5\frac{1}{3} - \frac{3}{2}$

4) $\frac{3}{4}$ of $\frac{7}{8}$ 5) $2\frac{2}{3} \times \frac{1}{4}$ 6) $1\frac{3}{4} \div \frac{5}{6}$

7) $5\frac{1}{3} \div 2\frac{1}{4}$

Solutions

1) $\frac{3}{4} + \frac{1}{3} = \frac{9}{12} + \frac{4}{12}$
$= \frac{13}{12}$
$= 1\frac{1}{12}$

2) $1\frac{5}{6} \div 2\frac{1}{2} = \frac{11}{6} \div \frac{5}{2}$
$= \frac{11}{6} + \frac{15}{6}$
$= \frac{26}{6}$
$= 4\frac{1}{3}$

3) $5\frac{1}{3} - \frac{3}{2} = \frac{16}{3} - \frac{3}{2}$
$= \frac{32}{6} - \frac{9}{6}$
$= \frac{23}{6}$
$= 3\frac{5}{6}$

4) $\frac{3}{4} \times \frac{7}{8} = \frac{21}{32}$

5) $2\frac{2}{3} \times \frac{1}{4} = \frac{\cancel{8}^{2}}{3} \times \frac{1}{\cancel{4}^{1}}$
$= \frac{2}{3}$

6) $1\frac{3}{4} \div \frac{5}{6} = \frac{7}{4} \div \frac{5}{6}$
$= \frac{7}{\cancel{4}^{2}} \times \frac{\cancel{6}^{3}}{5}$
$= \frac{21}{10}$
$= 2\frac{1}{10}$

7) $5\frac{1}{3} \div 2\frac{1}{4} = \frac{16}{3} \div \frac{9}{4}$
$= \frac{16}{3} \times \frac{4}{9}$
$= \frac{64}{27}$
$= 2\frac{10}{27}$

What are indices?

For the value 4^3, 4 is the base and 3 is the power or index (the plural is 'indices').

$4^3 = 4 \times 4 \times 4 = 64$
4 is multiplied by itself 3 times.

$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$
3 is multiplied by itself 5 times.

Fractional indices

If a number has a fractional index this means 'the root of'.

e.g. $64^{\frac{1}{3}} = \sqrt[3]{64}$ – this is the 3rd root of 64,
i.e. which number multiplied by itself 3 times gives the answer 64?

So $64^{\frac{1}{3}} = 4$ because $4 \times 4 \times 4 = 64$

$243^{\frac{1}{5}}$ means find the 5^{th} root of 243, i.e. find the number which when multiplied by itself 5 times gives the answer 243.

$243^{\frac{1}{5}} = 3$ because $3 \times 3 \times 3 \times 3 \times 3 = 243$

Clearly not all roots will have whole number values. Make sure you know how to use your calculator effectively when using indices, as calculators can vary.

Multiplication

$$3^4 \times 3^2 = (3 \times 3 \times 3 \times 3) \times (3 \times 3)$$
$$= 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$$

So $x^a \times x^b = x^{a+b}$

NOTE : This only works when you use the same base in your calculation.

So $2^4 \times 2^7 = 2^{(4+7)} = 2^{11}$ but you can't use this rule to work out e.g. $2^4 \times 3^5$

What happens with a multiplication such as $8a^5 \times 2a^6$?

The base is the same for both terms (base a) and multiplication is commutative, (it doesn't matter which order you do it in) so we can rewrite the expression as

$$8 \times a^5 \times 2 \times a^6$$

then rearrange this to give $8 \times 2 \times a^5 \times a^6$ to get $16a^{11}$

another example: $9b^3 \times 4b^7 = 36b^{10}$

What about $(x^a)^b$?

Consider $(4^3)^2$. *This is equal to* $4^3 \times 4^3$

$$= (4 \times 4 \times 4) \times (4 \times 4 \times 4)$$
$$= 4^6$$

$\boxed{\text{So } (x^a)^b = x^{ab}}$

Example: $(2^4)^3 = 2^{(4\times3)} = 2^{12}$

Division

We can see that $4^5 \div 4^3 = \dfrac{4\times4\times4\times4\times4}{4\times4\times4}$

Cancelling through gives us $= 4 \times 4 = 4^2$

$\boxed{\text{So } x^a \div x^b = x^{a-b}}$ *Again this rule only holds if the base is the same.*

What about a division such as $16c^5 \div 8c^3$?
Again, check that the base is the same then rewrite this as a fraction:

$$\frac{16c^5}{8c^3} = \frac{16}{8}\times\frac{c^5}{c^3}$$
$$= 2c^2$$

What do we mean by a number with the index 0?

Consider $4^3 \div 4^3 = 4^{3-3} = 4^0$
But $4^3 \div 4^3 = 1$ *so* $4^0 = 1$
Similarly $2^6 \div 2^6 = 2^0$ *but* $2^6 \div 2^6 = 1$

Generalising, $\boxed{x^0 = 1 \text{ for any value of } x}$

Negative indices

What do we mean by 4^{-3}?
The negative sign means 'one over' *so* 4^{-3} *means 'one over'* 4^3

i.e. $4^{-3} = \dfrac{1}{4^3} = \dfrac{1}{64}$

$$2^{-5} = \frac{1}{2^5} = \frac{1}{32}$$

$$\boxed{x^{-a} = \frac{1}{x^a}}$$

What does simplify mean?

Simplify means that you should try, as far as possible, to write the numbers in terms of one base with its index. Clearly this will not always be possible, so write the simplified expression in terms of the minimum number of bases possible.

e.g. Simplify $8^5 \times 8^8$

$8^5 \times 8^8 = \boxed{8^{13}}$

Simplify $a^9 \div a^4 \times b^3 \times b^4$

$a^9 \div a^4 \times b^3 \times b^4 = \boxed{a^5 b^7}$

Simplify $2^6 \div 4^2$

Initially you may think here that we cannot simplify the expression any further since the bases appear to be different.

Ask yourself the question — can either of these bases be written in terms of the other?

You should spot that $4 = 2^2$

So: $2^6 \div 4^2 = 2^6 \div (2^2)^2$
$= 2^6 \div 2^4$
$\boxed{= 2^2}$

And finally... $x^{\frac{a}{b}}$

$x^{\frac{a}{b}} = (x^{\frac{1}{b}})^a$ *which is the same as* $(x^a)^{\frac{1}{b}}$

Let's try finding $64^{\frac{2}{3}}$ using the two different methods above.

a) $64^{\frac{2}{3}} = (64^{\frac{1}{3}})^2$
$= 4^2$
$\boxed{= 16}$

b) $64^{\frac{2}{3}} = (64^2)^{\frac{1}{3}}$
$= 4096^{\frac{1}{3}}$
$\boxed{= 16}$

You should see that method a) is simpler in this case.

Laws of Indices

Examples:

Simplify the following expressions:

a) $4^{\frac{1}{2}}$ *b)* $64^{\frac{1}{3}}$

c) $27^{\frac{2}{3}}$ *d)* $a^{x+2} \times a^{2x}$

e) $2^{\frac{2}{3}} \times 32^{\frac{3}{5}}$ *f)* $(\frac{25}{4})^{\frac{-3}{2}}$

a) $4^{\frac{1}{2}} = \sqrt{4} = 2$

b) $64^{\frac{1}{3}} = \sqrt[3]{64} = 4$

c) $27^{\frac{2}{3}} = (27^{\frac{1}{3}})^2$
$= 3^2 = 9$

d) $a^{x+2} \times a^{2x} = a^{3x+2}$

e) $2^{\frac{2}{3}} \times 32^{\frac{3}{5}} = 2^{\frac{2}{3}} \times [(2^5)^{\frac{3}{5}}]$
$= 2^{\frac{2}{3}} \times 2^3$
$= 2^{3+\frac{2}{3}} = 2^{\frac{11}{3}}$

f) $(25/4)^{\frac{-3}{2}} = [(25/4)^{\frac{1}{2}}]^{-3}$
$= (5/2)^{-3}$
$= \frac{1}{(5/2)^3} = \frac{1}{125/8}$
$= \frac{8}{125}$

Now you have a go at these:

Express the following in their simplest form:

1) $9^{\frac{1}{2}}$ *2)* $81^{\frac{1}{4}}$ *3)* $64^{\frac{1}{6}}$ *4)* 27^0

5) $4^{-\frac{3}{2}}$ *6)* $64^{\frac{5}{6}}$ *7)* $64^{-\frac{2}{3}}$ *8)* $b^5 \times b^6$

9) $g^7 \div g^3$ *10)* $y^{10} \times y^2 \div y^5$

11) $(\frac{125}{8})^{\frac{1}{3}}$ *12)* $(\frac{16}{9})^{-\frac{3}{2}}$

Solutions

1) $9^{\frac{1}{2}} = 3$

2) $81^{\frac{1}{4}} = 3$

3) $64^{\frac{1}{6}} = 2$

4) $27^{0} = 1$

5) $4^{-\frac{3}{2}} = (4^{\frac{1}{2}})^{-3}$
$= 2^{-3}$
$= \frac{1}{2^3}$
$= \frac{1}{8}$

6) $64^{\frac{5}{6}} = (64^{\frac{1}{6}})^5$
$= 2^5$
$= 32$

7) $64^{-\frac{2}{3}} = (64^{-\frac{1}{3}})^2$
$= (\frac{1}{4})^2$
$= \frac{1}{16}$

8) $b^5 \times b^6 = b^{11}$

9) $g^7 \div g^3 = g^4$

10) $y^{10} \times y^2 \div y^5 = y^7$

11) $(\frac{125}{8})^{\frac{1}{3}} = \frac{5}{2}$

12) $(\frac{16}{9})^{-\frac{3}{2}} = [(\frac{16}{9})^{\frac{1}{2}}]^{-3}$
$= (\frac{4}{3})^{-3}$
$= \frac{1}{(4/3)^3}$
$= \frac{1}{64/27}$
$= \frac{27}{64}$

Factorising

Factorising means finding common aspects to each term in an expression. Basically, it's the opposite of expanding brackets, i.e. it's putting brackets in.

Make sure that after you have finished factorising, the contents of the brackets cannot be further factorised in any way.

Here are a few examples:

Factorise: 1) $14x + 21xy$

$\boxed{= 7x\,(2 + 3y)}$

Don't forget to check your answer by multiplying back out.

2) $2x^3y + 4xy^2$

If necessary, find common factors, by writing like this:

$2 \times x \times x \times x \times y + 4 \times x \times y \times y$ So $2x^3y + 4xy^2$ $\boxed{= 2xy\,(x^2 + 2y)}$

3) $4x^2 - 4x$

$\boxed{= 4x\,(x - 1)}$ [Notice the –1]

4) $3x\,(x + 2) - 4\,(x + 2)$

$(x + 2)$ is common to both expressions, so

$3x\,(x + 2) - 4\,(x + 2)$ $\boxed{= (x + 2)\,(3x - 4)}$

Now you have a go at some:

1) $9x - 21z$	2) $20x^2 - 4x$	3) $8x^2y + 28xy^2$
4) $3\pi a^2 + 4\pi ab$	5) $6 + 2x^2$	6) $y^3 + 3y^2 - y$
7) $4x\,(2x + 3) - 3\,(2x + 3)$		8) $5x^2\,(x - 1) - 2x\,(x - 1)$

1) $9x - 21z = 3\,(3x - 7z)$
2) $20x^2 - 4x = 4x\,(5x - 1)$
3) $8x^2y + 28xy^2 = 8 \times x \times x \times y + 28 \times x \times y \times y = 4xy\,(2x + 7y)$
4) $3\pi a^2 + 4\pi ab = \pi a\,(3a + 4b)$
5) $6 + 2x^2 = 2\,(3 + x^2)$
6) $y^3 + 3y^2 - y = y\,(y^2 + 3y - 1)$
7) $4x\,(2x + 3) - 3\,(2x + 3) = (2x + 3)\,(4x - 3)$
8) $5x^2\,(x - 1) - 2x\,(x - 1) = (x - 1)\,(5x^2 - 2x)$ — Notice the 2nd bracket can still be factorised, so this is not the final solution.

$= x\,(x - 1)\,(5x - 2)$ [It's usual to put this x at the front]

Algebraic Fractions

The four rules of algebraic fractions are exactly the same as those for numerical fractions.

Dodgy (i.e. incorrect!) cancelling does pose a problem in this section. Remember, you can only cancel when there is a multiplication sign between terms. (This multiplication sign may be implied.)

So, you can cancel here:

$$\frac{z^3}{z} = \frac{z^1 \times z \times z}{z^1} = z^2$$

and here:

$$\frac{\cancel{(z+3)}\ (z-2)}{\cancel{(z+3)}} = (z-2)$$

But, you cannot cancel here:

$$\frac{y^2 + 2y + 3}{y}$$

[Because of the addition signs]

If necessary, refer back to the four rules of numerical fractions at this point.

Now a few examples:

Simplify:

1) $\dfrac{3x+y}{2} - \dfrac{x-2y}{7}$

The lowest common denominator is 14, so:

$$\frac{3x+y}{2} - \frac{x-2y}{7} = \frac{7(3x+y)}{14} - \frac{2(x-2y)}{14}$$

$$= \frac{21x+7y-2x+4y}{14}$$

$$\boxed{= \frac{19x+11y}{14}}$$

2) $\dfrac{1}{x} + \dfrac{3}{x+1} = \dfrac{1(x+1)}{x(x+1)} + \dfrac{3x}{x(x+1)}$

$$= \frac{x+1+3x}{x(x+1)}$$

$$\boxed{= \frac{4x+1}{x(x+1)}}$$

3) $\dfrac{x+3}{2x} \times \dfrac{x^2}{(x+3)^2}$

We can cancel:

$$\frac{\cancel{x+3}}{2\cancel{x}} \times \frac{x^{\cancel{2}}}{(x+3)^{\cancel{2}}} \qquad \boxed{= \frac{x}{2(x+3)}}$$

Algebraic Fractions

4) $\dfrac{a(a-b)}{b^2} \div \dfrac{(a-b)}{a}$

$= \dfrac{a\cancel{(a-b)}}{b^2} \times \dfrac{a}{\cancel{(a-b)}}$

$\boxed{= \dfrac{a^2}{b^2}}$

5) *Solve this equation:*

$\dfrac{1}{4} + \dfrac{1}{x+3} = 2$

To do this you need the LHS to be a single fraction:

$\dfrac{1(x+3)}{4(x+3)} + \dfrac{4}{4(x+3)} = 2$

$\dfrac{x+7}{4(x+3)} = 2$

$x + 7 = 8(x + 3)$

$x + 7 = 8x + 24$

$-17 = 7x$

$\boxed{x = \dfrac{-17}{7}}$

Now you try these:

1) *Cancel these fractions as far as possible:*

a) $\dfrac{3x^2}{7x}$ b) $\dfrac{8x^2\ (x+3)}{4x}$ c) $\dfrac{8x+16}{2x-4}$

2) *Express as a single fraction:*

a) $3 + \dfrac{2}{x}$ b) $\dfrac{1}{x+1} - \dfrac{3}{x-2}$ c) $\dfrac{a}{b} - \dfrac{2a}{3b}$

3) *Simplify these expressions:*

a) $\dfrac{3x+9}{4} \times \dfrac{x}{3(x+3)}$ b) $\dfrac{x+3}{x^2} \times \dfrac{x}{4}$

c) $\dfrac{x(x-3)}{3} \div \dfrac{x-3}{x}$ d) $12x(x+2) \div \dfrac{3x+6}{x}$

Algebraic Fractions

1) a) $\dfrac{3x^2}{7\cancel{x}} = \dfrac{3x}{7}$ b) $\dfrac{{}_2\cancel{8}x^2\ (x+3)}{{}_1\cancel{4}\cancel{x}} = 2x(x+3)$

c) $\dfrac{{}_4\cancel{8}(x+2)}{{}_1\cancel{2}(x-2)} = \dfrac{4(x+2)}{(x-2)}$

---

2) a) $3 + \dfrac{2}{x} = \dfrac{3x}{x} + \dfrac{2}{x}$

$= \dfrac{3x+2}{x}$

b) $\dfrac{1}{x+1} - \dfrac{3}{x-2} = \dfrac{1(x-2)}{(x+1)(x-2)} - \dfrac{3(x+1)}{(x+1)(x-2)}$

$= \dfrac{x-2-3x-3}{(x+1)(x-2)}$

$= \dfrac{-2x-5}{(x+1)(x-2)}$

c) $\dfrac{a}{b} - \dfrac{2a}{3b} = \dfrac{3a}{3b} - \dfrac{2a}{3b}$

$= \dfrac{a}{3b}$

---

3) a) $\dfrac{3x+9}{4} \times \dfrac{x}{3(x+3)} = \dfrac{\cancel{3(x+3)}}{\cancel{4}} \times \dfrac{x}{\cancel{3(x+3)}}$

$= \dfrac{x}{4}$

b) $\dfrac{x+3}{x^2} \times \dfrac{\cancel{x}}{4} = \dfrac{x+3}{4x}$

c) $\dfrac{x(x-3)}{3} \div \dfrac{x-3}{x} = \dfrac{x\cancel{(x-3)}}{3} \times \dfrac{x}{\cancel{x-3}}$

$= \dfrac{x^2}{3}$

d) $12x(x+2) \div \dfrac{3x+6}{x} = 12x\,(x+2) \times \dfrac{x}{3x+6}$

$= {}^{4}\cancel{12}x\cancel{(x+2)} \times \dfrac{x}{{}_1\cancel{3}\cancel{(x+2)}}$

$= 4x^2$

Changing the Subject of a Formula

In the formula $y = mx + c$, y is known as the __subject__. If you wanted to make x the subject of the formula, you would simply have to rearrange it, just like when solving linear equations:

First take c from both sides: $y - c = mx$

Then divide both sides by m: $\dfrac{y - c}{m} = x$ *Notice the whole of the left hand side (LHS) must be divided by m*

It doesn't matter whether the subject of the formula ends up on the right hand side (RHS) or LHS

Now, let's look at a few examples:

1) Make b the subject of the formula:

$$a = \frac{b^2}{2}$$

$$2a = b^2 \qquad \boxed{b = \pm\sqrt{(2a)}}$$

The subject of the Formula One.

Sometimes you need to gather all the terms involving the proposed subject on one side of the equation, first:

2) Make a the subject of the formula:

$$a^2 + b^2 = (3 + a)(a - b)$$

$$a^2 + b^2 = 3a - 3b + a^2 - ab$$

Gather all terms involving a on the LHS

$$a^2 + b^2 - 3a - a^2 + ab = -3b$$

Gather all other terms on the RHS

$$\cancel{a^2} - 3a - \cancel{a^2} + ab = -b^2 - 3b$$

Take out any common factors

$$a(b - 3) = -b(b + 3)$$

Divide both sides by (b – 3)

$$\boxed{a = \frac{-b(b+3)}{b-3}}$$

3) Make v the subject:

$$\frac{1}{p} = \frac{1}{v} + \frac{1}{t}$$

First make the RHS a single fraction

$$\frac{1}{p} = \frac{t}{vt} + \frac{v}{vt}$$

$$\frac{1}{p} = \frac{t + v}{vt}$$

Now multiply both sides by vt, then both sides by p.
(This can all be done in one step).

$$vt = p(t + v)$$

$$vt = pt + pv$$

$$vt - pv = pt$$

$$v(t - p) = pt$$

$$\boxed{v = \frac{pt}{t - p}}$$

Changing the Subject of a Formula

Try these for yourself:

1) $x = 5y^2$. Find y in terms of x, if y is a positive number.
2) Express a in terms of b, given that: $b(a + 2) = 4$
3) Make C the subject of the formula: $F = \frac{9}{5}C + 32$
4) Make z the subject of the formula: $\frac{z+1}{z+4} = \frac{z+2}{z+3}$
5) Make t the subject of the formula: $s = \frac{\sqrt{(t+u)}}{u}$
6) Make x the subject of the formula: $y = 3\sqrt{x+2}$

1) $x = 5y^2$

$\frac{x}{5} = y^2$

$y = \sqrt{\frac{x}{5}}$

2) $b(a + 2) = 4$

$ab + 2b = 4$

$ab = 4 - 2b$ $\qquad a = \frac{4 - 2b}{b}$

3) $F = \frac{9}{5}C + 32$

$F - 32 = \frac{9}{5}C$

$5(F - 32) = 9C$

$\frac{5}{9}(F - 32) = C$

4) $\frac{z+1}{z+4} = \frac{z+2}{z+3}$

$(z + 1)(z + 3) = (z + 4)(z + 2)$

$z^2 + 4z + 3 = z^2 + 6z + 8$

$3 - 8 = \cancel{z^2} + 6z - \cancel{z^2} - 4z$

$-5 = 2z$ $\qquad z = \frac{-5}{2}$

5) $s = \frac{\sqrt{(t+u)}}{u}$

$su = \sqrt{(t+u)}$

$s^2u^2 = t + u$

$t = s^2u^2 - u$ $\qquad t = u(s^2u - 1)$

6) $y = 3\sqrt{x+2}$

$\frac{y}{3} = \sqrt{x+2}$

$\frac{y^2}{9} = \frac{x}{2}$

$2y^2 = 9x$ $\qquad x = \frac{2y^2}{9}$

Quadratic Equations

$y = x^2 + 2x + 1$ is a quadratic equation.

Quadratic equations must have an x^2 term in them but can also have an x term and/or a number term (a constant). To be a quadratic equation they can not have any other powers of x such as x^3 or $x^{\frac{1}{3}}$.

The following expressions are quadratics:

$y = x^2 + 4$ $\quad$ $y = 4x^2 - 2x - 3$ $\quad$ $y = x^2 + 7x$

The following are not quadratics:

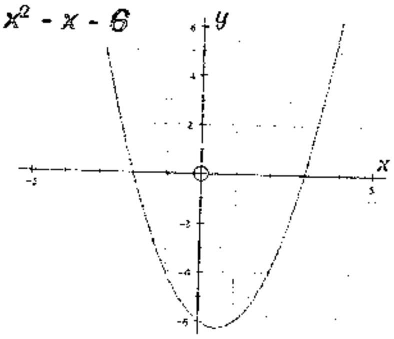
$y = x^2 + 2x^3 + 1$ $\quad$ $y = x^2 + 2x + x^{-2}$

Solving quadratic equations

What do we mean by solving quadratic equations?

This is the process by which you find the value(s) of x for which $y = 0$.
This can be demonstrated clearly on a graph.

$y = x^2 - x - 6$



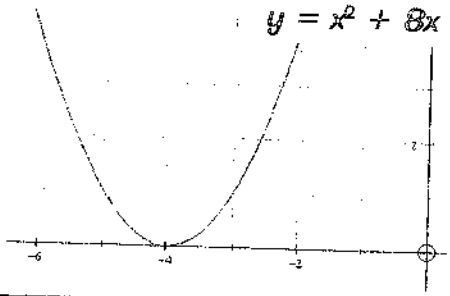
The two solutions of the equation $y = x^2 - x - 6$ are the values of x at which $y = 0$.

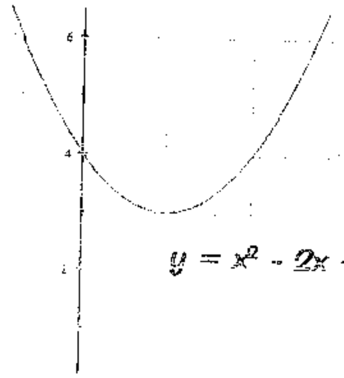
These values occur when the curve intersects the x-axis.
In this example you can see that these are when $\boxed{x = -2}$ and $\boxed{x = 3}$

In the case opposite there is only one solution of the equation $y = x^2 + 8x + 16$ since the curve is tangential to the x-axis. It touches at one point without crossing the axis.

The solution is $\boxed{x = -4}$

$y = x^2 + 8x + 16$





$y = x^2 - 2x + 4$

In this case the curve $y = x^2 - 2x + 4$ does not intersect the x-axis, so there are no real solutions to the equation.

Factorising $y = ax^2 + bx + c$

Now we've seen what is meant by solving quadratic equations, how do we actually do it?
There are 3 methods:

1. *Factorisation*
2. *Completing the square*
3. *Using the quadratic formula*

Methods 2 and 3 will be covered in detail in the P1 module

Factorising is easier if $a = 1$

Factorising means putting the expression into brackets. This is easier when the number in front of the x^2 term is 1, i.e. $a = 1$. We'll start with this kind.

First of all, make sure that you have the x^2, x and constant terms on the same side of the equation. This now gives you

$$y = x^2 + bx + c$$

Find 2 values which give the value 'c' when multiplied together.

The same 2 values, must give the value 'b' when added together.

Here's an example.

$$\text{Solve} \quad y = x^2 + 7x + 12$$

In this case $b = 7$ and $c = 12$.

Put the expression equal to zero since when you solve equations, $y = 0$

When you first start doing these it can be useful to table the pairs of numbers which multiply to give 'c' and the sum of these numbers.

Numbers which multiply to give 12	These numbers added
1×12	$1 + 12 = 13$
2×6	$2 + 6 = 8$
3×4	$3 + 4 = 7$

You'll see that 3 and 4 multiply to give 12 and add to give 7 so we can put the expression into brackets: $(x + 3)(x + 4) = 0$ *(Check this by multiplying out the brackets.)*

BUT! You haven't finished yet!

If $(x + 3)(x + 4) = 0$ *then either* $x + 3 = 0$ *or* $x + 4 = 0$

So $\boxed{x = -3}$ *or* $\boxed{x = -4}$ *SOLVED!*

Factorising $y = ax^2 + bx + c$

Solve $y = x^2 + 9x + 20$

$b = 9$ and $c = 20$

Numbers which multiply to give 20	These numbers added
1×20	$1 + 20 = \quad 21$
2×10	$2 + 10 = \quad 12$
4×5	$4 + 5 = \quad 9$

$(x + 4)(x + 5) = 0$ so $\boxed{x = -4}$ or $\boxed{x = -5}$

Solve $y = x^2 + 5x - 14$

$b = 5$ and $c = -14$

Numbers which multiply to give -14	These numbers added
-1×14	$-1 + 14 = \quad 13$
-2×7	$-2 + 7 = \quad 5$
1×-14	$1 + (-14) = \quad -13$
2×-7	$2 + (-7) = \quad -5$

$(x - 2)(x + 7) = 0$ so $\boxed{x = 2}$ or $\boxed{x = -7}$

Obviously you don't need to work out all the values if you get the right pairing. Also, a tip for you — if 'c' is a __negative__ value then __one__ value of x must be __negative__ and the __other__ must be __positive__; so you know your brackets will be:

$$(x + ?)(x - ?) = 0$$

Solve $y = x^2 - 8x + 15$

$b = -8$ and $c = 15$

Numbers which multiply to give 15	These numbers added
1×15	$1 + 15 = \quad 16$
3×5	$3 + 5 = \quad 8$
-1×-15	$-1 + (-15) = \quad -16$
-3×-5	$-3 + (-5) = \quad -8$

$(x - 3)(x - 5) = 0$ so $\boxed{x = 3}$ or $\boxed{x = 5}$

Another tip for you — if 'c' is a __positive__ value but 'b' is a __negative__ value then your brackets will b

$$(x - ?)(x - ?) = 0$$

so you just need __negative__ values in your table.

Factorising $y = ax^2 + bx + c$ when $a \neq 1$

This is a bit more complicated. Sometimes it's just worth trying different values in the brackets but there is a more 'organised' way which you may prefer when you're starting off.

Here's an example.

$$y = 2x^2 + 7x + 6$$

1) First of all try to establish which terms go at the start of each bracket. Since you have $2x^2$ here, the brackets must be: $(2x + ?)(x + ?)$

2) Now draw a table showing the 2 numbers that multiply to give 'c'

x value	Pairs of numbers which multiply to give 6			
2x	1	6	3	2
x	6	1	2	3

Multiply these diagonally with the x values as shown above then add the 2 results:

$(2x \times 6) + (x \times 1) = 13x$

$(2x \times 1) + (x \times 6) = 8x$

$(2x \times 2) + (x \times 3) = 7x$

$(2x \times 3) + (x \times 2) = 8x$

you can see that this is the correct pairing

3) The solution is given along the rows $(2x + 3)(x + 2) = 0$

either $(2x + 3) = 0$ so $\boxed{x = \frac{-3}{2}}$ or $(x + 2) = 0$ so $\boxed{x = -2}$

Here's another example:

$$y = 3x^2 + x - 10$$

The brackets must be $(3x \pm ?)(x \pm ?)$

Table the possible pairings — you get:

x value	Pairs of numbers which multiply to give (-10)							
3x	-10	10	-1	1	-5	5	-2	2
x	1	-1	10	-10	2	-2	5	-5

There's no need to write down all the possible diagonal multiplications — you should be able to do these in your head to save time.

You should see that:

$$(3x \times 2) + (x \times -5) = x$$

so, reading along the row: $(3x - 5)(x + 2) = 0$

Either $(3x - 5) = 0$ so $\boxed{x = \frac{5}{3}}$ or $(x + 2) = 0$ so $\boxed{x = -2}$

The Difference of Two Squares

Whenever we have

" Y = a squared term MINUS another squared term "

This is called: THE DIFFERENCE OF TWO SQUARES

e.g. $y = a^2 - b^2$

Why is this so important?

Well, it's because if you recognise that an expression is 'the difference of two squares' you can factorise it very easily.

If $y = a^2 - b^2$ then factorising gives us: $y = (a + b)(a - b)$

e.g. Factorise the following expressions:

a) $y = x^2 - 25$

$\boxed{y = (x + 5)(x - 5)}$

b) $y = 4x^2 - 81$

$\boxed{y = (2x + 9)(2x - 9)}$

Questions on Quadratic Equations

Solve the following quadratic equations:

1) $x^2 + 3x + 2 = 0$
2) $x^2 + 8x + 7 = 0$
3) $x^2 + 4x - 12 = 0$
4) $x^2 - 4x - 12 = 0$
5) $x^2 - 7x + 10 = 0$
6) $x^2 - 14x + 40 = 0$
7) $2x^2 + 9x + 9 = 0$
8) $5x^2 + 13x + 6 = 0$
9) $2x^2 - x - 10 = 0$
10) $3x^2 - 16x + 21 = 0$

Factorise the following expressions:

1) $y = x^2 - 9$
2) $y = x^2 - 16$
3) $y = 4x^2 - 16$
4) $y = 9x^2 - 25$

The difference of two squares

1) $y = x^2 - 9$

   $y = (x + 3)(x - 3)$

2) $y = x^2 - 16$

   $y = (x + 4)(x - 4)$

3) $y = 4x^2 - 16$

   $y = (2x + 4)(2x - 4)$

4) $y = 9x^2 - 25$

   $y = (3x + 5)(3x - 5)$

Solving quadratic equations

1) $x^2 + 3x + 2 = 0$

   $(x + 2)(x + 1) = 0$

   $x = -2$ or $x = -1$

2) $x^2 + 8x + 7 = 0$

   $(x + 7)(x + 1) = 0$

   $x = -7$ or $x = -1$

3) $x^2 + 4x - 12 = 0$

   $(x - 2)(x + 6) = 0$

   $x = 2$ or $x = -6$

4) $x^2 - 4x - 12 = 0$

   $(x + 2)(x - 6) = 0$

   $x = -2$ or $x = 6$

5) $x^2 - 7x + 10 = 0$

   $(x - 2)(x - 5) = 0$

   $x = 2$ or $x = 5$

6) $x^2 - 14x + 40 = 0$

   $(x - 4)(x - 10) = 0$

   $x = 4$ or $x = 10$

7) $2x^2 + 9x + 9 = 0$

   $(2x + 3)(x + 3) = 0$

   $x = \frac{-3}{2}$ or $x = -3$

8) $5x^2 + 13x + 6 = 0$

   $(5x + 3)(x + 2) = 0$

   $x = \frac{-3}{5}$ or $x = -2$

9) $2x^2 - x - 10 = 0$

   $(2x - 5)(x + 2) = 0$

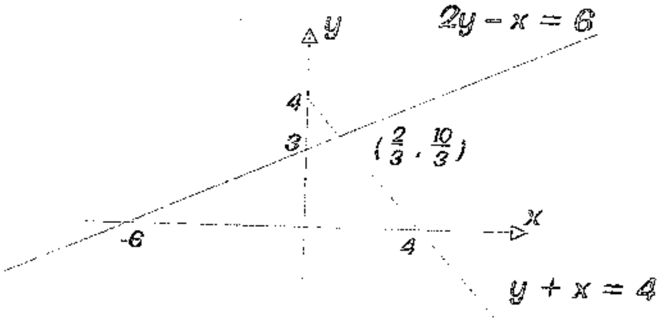
   $x = \frac{5}{2}$ or $x = -2$

10) $3x^2 - 16x + 21 = 0$

   $(3x - 7)(x - 3) = 0$

   $x = \frac{7}{3}$ or $x = 3$

The Elimination Method



The graph displays two linear equations. The p[oint] of intersection represents the only value of x [and] only value of y, which satisfies both equation[s]. $x = \frac{2}{3}$ and $y = \frac{10}{3}$ is the solution of the pair of simultaneous equations:

$$2y - x = 6$$
$$y + x = 4$$

Solving Equations Algebraically

Although simultaneous equations can be solved graphically, it's more efficient to solve them algebraically.

The <u>elimination</u> method relies on one variable being removed, to allow us to find the sole value of the other variable.

Here's an example of the elimination method:

Solve: $3x - 2y = 1$ $\quad \underline{1}$

$\qquad\qquad 2x + 3y = 11.5$ $\quad \underline{2}$

In order to eliminate a variable, you need the same coefficient of that variable in each equati[on] (ignoring the sign). Suppose you choose to eliminate y:

There are 2 y's in the first equation and 3 y's in the second. The lowest common multiple of 2 and 3 is 6, so we require 6 y's in each equation. To achieve this we need to multiply the first equation by 3 and the second equation by 2. It's perfectly acceptable to multiply equations by a constant, provided every term in the equation is transformed in the same way

So:	$3x - 2y = 1$
	$2x + 3y = 11.5$
$3 \times \underline{1}$	$9x - 6y = 3$
$2 \times \underline{2}$	$4x + 6y = 23$

We're eliminating y. Notice we have a positive and a negative coefficient of y. Because the signs are different, we eliminate by adding the equations together. If necessary, look at the variable you are eliminating and remember:

<u>S</u>igns the <u>S</u>ame <u>S</u>o <u>S</u>ubtract

$\boxed{\underline{1} + \underline{2} \qquad 13x = 26}$ So $\boxed{x = 2}$

Now we have found the value of x, we can use this to work out y:

Substitute x in $\underline{2}$: (Choose $\underline{2}$ to avoid the negative 2y).

$$(2 \times 2) + 3y = 11.5$$
$$4 + 3y = 11.5$$
$$3y = 7.5$$

So $\boxed{y = 2.5}$

You can now perform a check:

Substitute x and y in $\underline{1}$: $(3 \times 2) - (2 \times 2.5) = 1$ ✓

The Substitution Method

Another method of algebraically solving simultaneous equations is the substitution method.
This involves rearranging one of the equations and substituting it into the other.

Looking back at the graphical example on the last page:

$$2y - x = 6 \qquad \underline{1}$$
$$y + x = 4 \qquad \underline{2}$$

Rearranging [illegible], gives:

$$x = 4 - y$$

Now replace the x in equation [illegible], by $4 - y$

$$2y - (4 - y) = 6$$
$$2y - 4 + y = 6$$
$$3y - 4 = 6$$
$$3y = 10$$

$$\boxed{y = \frac{10}{3}}$$

Substitute y in [illegible]:

$$\frac{10}{3} + x = 4$$

$$\boxed{x = \frac{2}{3}}$$

This substitution method is used in the AS course, as it proves particularly useful when one equation is linear and the other is quadratic.

Now your turn...

1) Solve these simultaneous equations, by a method of elimination:

a) $5x + 3y = 17$
   $4x + 10y = 25$

b) $7x - 3y = 48$
   $x + 0.5y = 5$

c) $5p + 2q = -30$
   $3p + 4q = -32$

2) Solve these simultaneous equations, by a method of substitution:

a) $x = 2y + 1$
   $3x - 4y = 7$

b) $a - 3b = 11$
   $5a + 2b = 4$

c) $3x + y = 7$
   $x - 2.5y = 8$

3) In a quiz, 1 correct answer and 3 incorrect answers scores 6 points, whilst 2 correct and 4 incorrect scores 16 points.

a) What is the value of a correct answer?

b) How many points are deducted for an incorrect answer?

1)a) $5x + 3y = 17$ ①

$4x + 10y = 25$ ②

4 x ① $20x + 12y = 68$ ③

5 x ② $20x + 50y = 125$ ④

④ − ③ $38y = 57$ so $\underline{y = 1.5}$

Sub y in ①

$5x + (3 \times 1.5) = 17$

$5x = 12.5$

$\underline{x = 2.5}$

b) $7x - 3y = 48$ ①

$x + 0.5y = 5$ ②

6 x ② $6x + 3y = 30$ ③

① + ③ $13x = 78$

$\underline{x = 6}$

Sub x in ②

$6 + 0.5y = 5$

$0.5y = -1$

$\underline{y = -2}$

c) $5p + 2q = -30$ ①

$3p + 4q = -32$ ②

2 x ① $10p + 4q = -60$ ③

③ − ② $7p = -28$

$\underline{p = -4}$

Sub p in ①

$(5 \times -4) + 2q = -30$

$2q = -10$

$\underline{q = -5}$

2)a) $x = 2y + 1$ ①

$3x - 4y = 7$ ②

Sub ① in ②

$3(2y + 1) - 4y = 7$

$6y + 3 - 4y = 7$

$2y + 3 = 7$ so $\underline{y = 2}$

Sub y in ①

$x = (2 \times 2) + 1$

$\underline{x = 5}$

b) $a - 3b = 11$ ①

$5a + 2b = 4$ ②

From ① $a = 11 + 3b$ ③

Sub ③ into ②

$5(11 + 3b) + 2b = 4$

$55 + 15b + 2b = 4$

$17b + 55 = 4$ $\underline{b = -3}$

Sub b in ①

$a - (3 \times -3) = 11$

$\underline{a = 2}$

c) $3x + y = 7$ ①

$x - 2.5y = 8$ ②

From ① $y = 7 - 3x$ ③

Sub ③ into ②

$x - 2.5(7 - 3x) = 8$

$x - 17.5 + 7.5x = 8$

$8.5x - 17.5 = 8$ $\underline{x = 3}$

Sub x in ①

$(3 \times 3) + y = 7$

$\underline{y = -2}$

3) Let the value of a correct answer be C and the value of an incorrect answer be W.

$C + 3W = 6$ ①

$2C + 4W = 16$ ②

2 x ① $2C + 6W = 12$ ③

③ − ② $2W = -4$

$\underline{W = -2}$

Sub W in ①

$C + (3 \times -2) = 6$

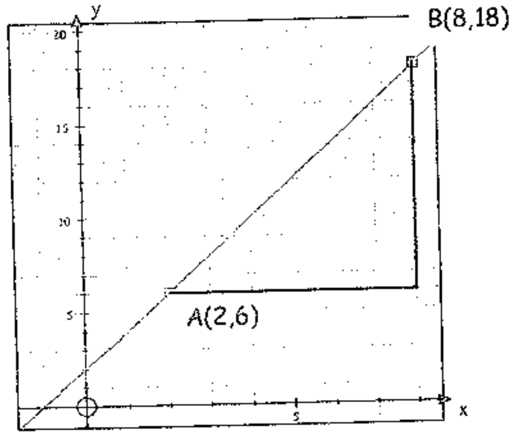
$\underline{C = 12}$

So, a) The value of a correct answer is 12 points.

b) 2 points are deducted for an incorrect answer.

Finding the Gradient

To find the gradient of a straight line we need to choose 2 points on the line.
In this case we have point A(2,6) and point B(8,18).



The gradient $= \dfrac{18-6}{8-2} = \dfrac{12}{6} = 2$

OR:- $\dfrac{6-18}{2-8} = \dfrac{-12}{-6} = 2$

So the gradient $= \dfrac{\text{change in } y}{\text{change in } x}$

***MAKE SURE** that you __always__ position the y-coordinate of a given point __over__ the x-coordinate of that point in your fraction. It __doesn't matter__ which y-coordinate from the two points selected goes first so long as the corresponding x-coordinate is positioned __beneath__ it.*

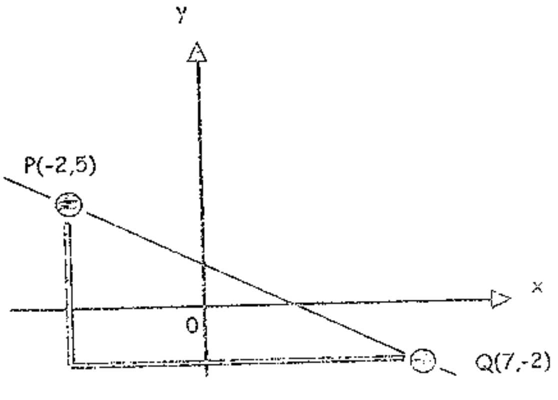
y-coordinates

Both coordinates for point B $\dfrac{18 - 6}{8 - 2}$ *Both coordinates for point A*

x-coordinates

It's always useful to draw a sketch for gradient questions.
Also watch out for those negative values as they can cause problems.

E.g. Find the gradient of the line through the points (-2,5) and (7,-2)



Gradient $= \dfrac{\text{change in } y}{\text{change in } x}$

$= \dfrac{5-(-2)}{(-2)-7}$

$= \dfrac{5+2}{-9}$

$\boxed{= \dfrac{-7}{9}}$

Finding the Gradient

Now you have a go at these.

Find the gradients of the lines which pass through the following points:

1. (5,6) and (9,15)
2. (2,12) and (4,1)
3. (-6,-2) and (3,4)
4. (-4,8) and (10,-3)

1. gradient $= \dfrac{\text{change in } y}{\text{change in } x}$
$= \dfrac{15 - 6}{9 - 5}$
$= \dfrac{9}{4}$

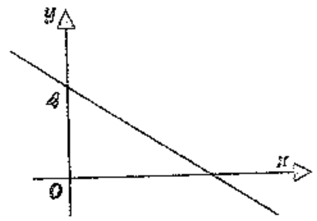
2. gradient $= \dfrac{\text{change in } y}{\text{change in } x}$
$= \dfrac{12 - 1}{2 - 4}$
$= \dfrac{11}{-2}$ which is the same as $\dfrac{-11}{2}$

3. gradient $= \dfrac{\text{change in } y}{\text{change in } x}$
$= \dfrac{4-(-2)}{3-(-6)}$
$= \dfrac{4 + 2}{3 + 6}$
$= \dfrac{6}{9}$ which is the same as $\dfrac{2}{3}$

4. gradient $= \dfrac{\text{change in } y}{\text{change in } x}$
$= \dfrac{8 - (-3)}{-4 - 10}$
$= \dfrac{8 + 3}{-4 - 10}$
$= \dfrac{-11}{14}$

y-intercept, Horizontal and Vertical Lines

The y-intercept

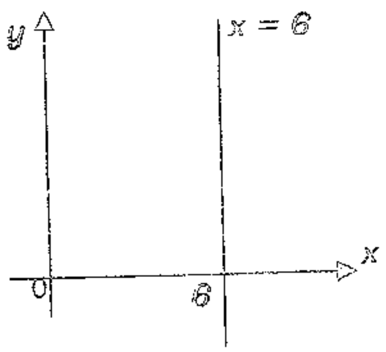


The y-intercept is the y-coordinate of the point at which a line intersects the y-axis; i.e. where $x = 0$.

In this case the y-intercept = 4

Equations of Vertical and Horizontal Lines

Remember $x = ?$ is vertical, and $y = ?$ is horizontal.



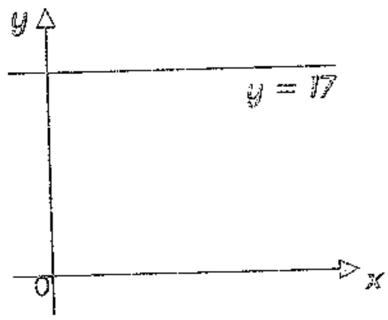
Whenever you have a __vertical__ line then the equation of the line will always be:

$$x = ?$$

where ? is the point at which the line intersects the x-axis.

In this example $x = 6$.

This is because along the line, y can take __any__ value but the value of x is __always 6__.



Whenever you have a __horizontal__ line then the equation of the line will always be:

$$y = ?$$

where ? is the point at which the line intersects the y-axis.

In this example $y = 17$.

This is because along the line, x can take __any__ value but the value of y is __always 17__.

$y=mx + c$

Equation of A Straight Line in the Form $y=mx + c$

The equation of a straight line can be written in the form:

$$\boxed{y = mx + c}$$

m is the gradient

c is the y-intercept

If you know the gradient and the y-intercept of a straight line, you can give the equation of that line

e.g. *A line has a gradient = 4 and y-intercept = 6. Find the equation of the line.*

$$y = mx + c$$

$$\boxed{y = 4x+6}$$

e.g. Find the equation of the line which has gradient = 2 and which passes through the point (0,7).

"What is the y-intercept?", you might think. The y-intercept is the point at which the line intersects the y-axis, i.e. when x = 0. So the y-intercept = 7.

$$y = mx + c$$

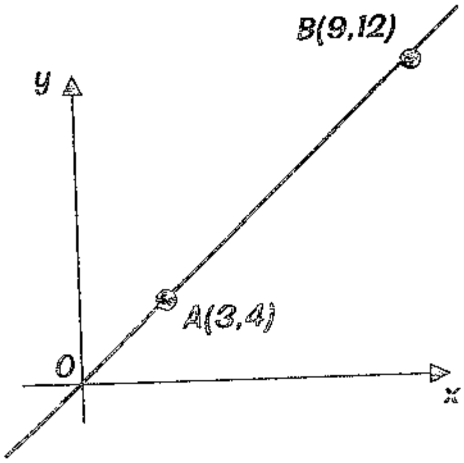
$$\boxed{y = 2x + 7}$$

Now you try some:

1. *Find the equation of the line with gradient 3 and y-intercept 5.*
2. *Find the equation of the line which has y-intercept -2 and gradient 4.*
3. *Find the equation of the line which has gradient -2 and which passes through the point (0,6).*

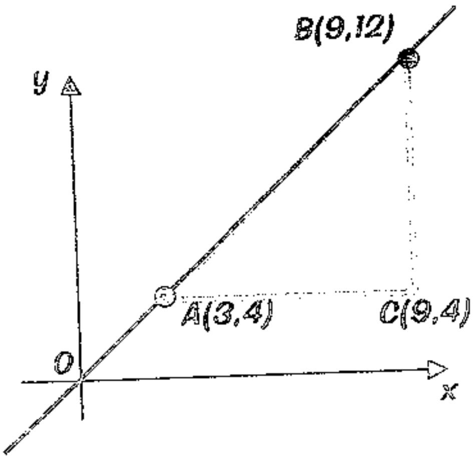
1. $y = 3x + 5$
2. $y = 4x - 2$
3. $y = -2x + 6$

Finding the Distance Between 2 Points



How can we find the distance between points A and B?

Start by sketching the line.



Complete the right-angled triangle ABC.
You should be able to see that point C has coordinates (9,4).
Find the distances AC and BC.

$AC = 9 - 3 = 6$
$BC = 12 - 4 = 8$

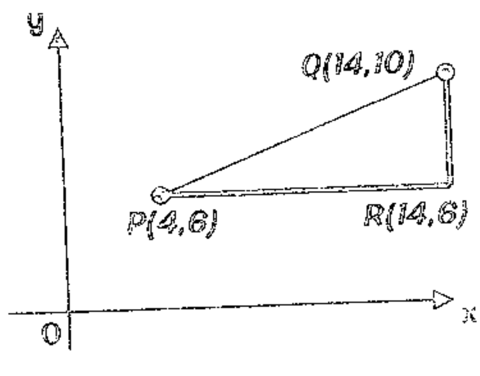
We can now use Pythagoras' theorem.

By Pythagoras' theorem
$AB^2 = AC^2 + BC^2$
$AB^2 = 6^2 + 8^2$
$AB^2 = 36 + 64$

$\boxed{AB = \sqrt{100} = 10}$

Example:

Find the distance between points P(4,6) and Q(14,10).



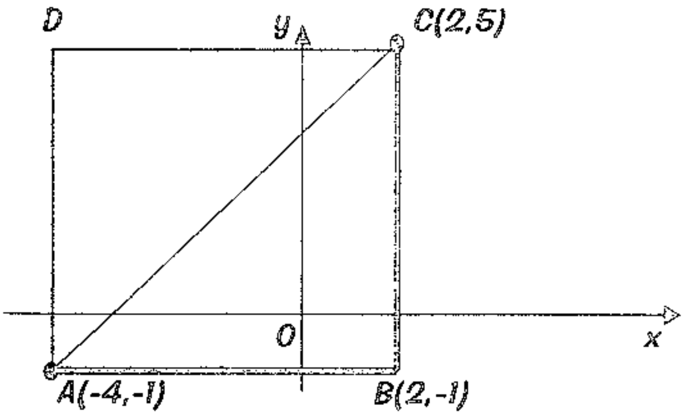
Draw a sketch marking on the points P and Q.
Complete the right angled triangle PQR.
$PR = 14 - 4 = 10$
$QR = 10 - 6 = 4$

By Pythagoras's theorem
$PQ^2 = PR^2 + QR^2$
$PQ^2 = 10^2 + 4^2$
$PQ^2 = 100 + 16$
$PQ^2 = 116$
$\boxed{PQ = \sqrt{116}}$

Finding the Distance Between 2 Points

Example

Points A(-4,-1) and C(2,5) are opposite vertices of a square.
Find the lengths of the sides of the square and the length of the diagonals.



AB is one side of the square
The coordinates of B are (2,-1)

$AB = 2 - (-4)$
$AB = 2 + 4$
$AB = 6$

The sides of the square are 6 units long

AC is one of the diagonals

By Pythagoras's theorem
$AC^2 = AB^2 + BC^2$
$AB^2 = 6^2 + 6^2$
$AB^2 = 36 + 36$
$AB = \sqrt{72}$

So the diagonals are $\sqrt{72}$ units long

Now you have a go at these:

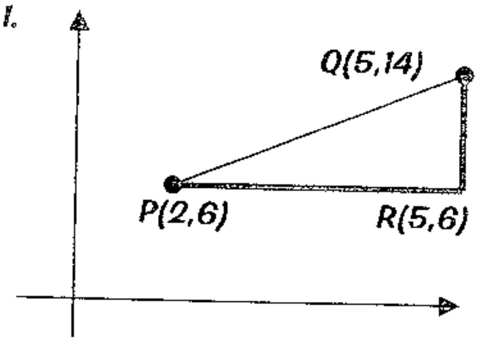
1. Find the distance between points P(2,6) and Q(5,14)

2. Find the lengths of the each of the sides of the triangle with vertices at points A(3,5), B(7,2) and C(-1,3) respectively.

3. Find the lengths of the sides of triangle PQR which has vertices at points P(-4,2), point Q(-1,6) and point R(3,3). What kind of triangle is this?

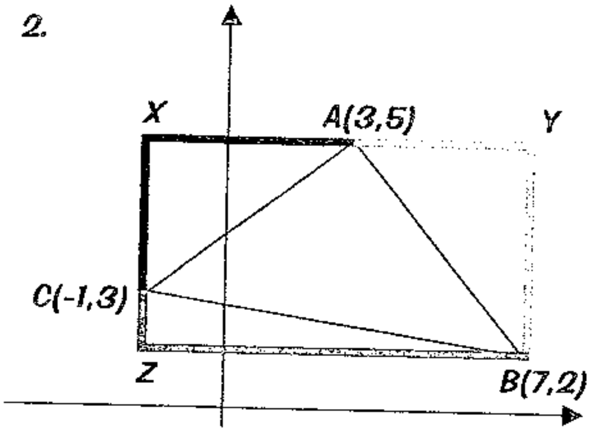
Solutions

1.



$$
\begin{aligned}
PQ^2 &= PR^2 + QR^2 \\
&= (5-2)^2 + (14-6)^2 \\
&= 3^2 + 8^2 \\
&= 9 + 64 \\
PQ &= \sqrt{73}
\end{aligned}
$$

2.



Complete the three right-angled triangles and label the points.

In Δ AYB

$$
\begin{aligned}
AB^2 &= AY^2 + BY^2 \\
&= (7-3)^2 + (5-2)^2 \\
&= 4^2 + (-3)^2 \\
AB &= \sqrt{16+9} \\
AB &= 5
\end{aligned}
$$

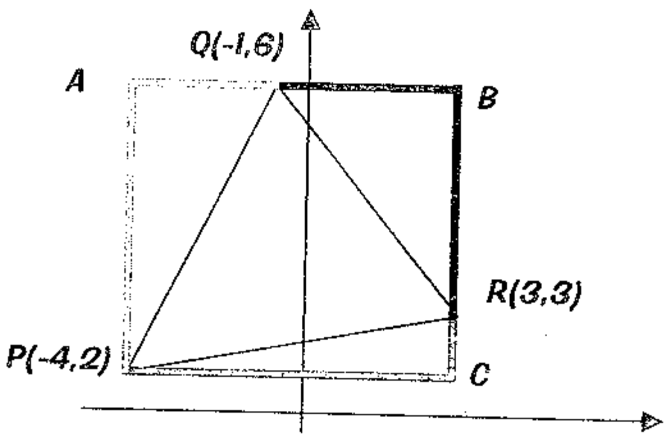
In Δ ACX

$$
\begin{aligned}
AC^2 &= AX^2 + CX^2 \\
&= (3-(-1))^2 + (5-3)^2 \\
&= 4^2 + 2^2 \\
AC &= \sqrt{16+4} \\
AC &= \sqrt{20}
\end{aligned}
$$

In Δ BCZ

$$
\begin{aligned}
BC^2 &= BZ^2 + CZ^2 \\
&= (7-(-1))^2 + (2-3)^2 \\
&= 8^2 + (-1)^2 \\
BC &= \sqrt{64+1} \\
BC &= \sqrt{65}
\end{aligned}
$$

3.



In ΔAPQ

$$
\begin{aligned}
PQ^2 &= AP^2 + AQ^2 \\
&= (6-2)^2 + (-4-(-1))^2 \\
&= 4^2 + (-3)^2 \\
PQ &= \sqrt{25} = 5
\end{aligned}
$$

In ΔBQR

$$
\begin{aligned}
QR^2 &= BQ^2 + BR^2 \\
&= (3-(-1))^2 + (6-3)^2 \\
&= 4^2 + 3^2 \\
QR &= \sqrt{25} = 5
\end{aligned}
$$

In ΔCPR

$$
\begin{aligned}
PR^2 &= PC^2 + CR^2 \\
&= (3-(-4))^2 + (3-2)^2 \\
&= 7^2 + 1^2 \\
PR &= \sqrt{50}
\end{aligned}
$$

Since PQ and QR are equal in length, ΔPQR must be an isosceles triangle.

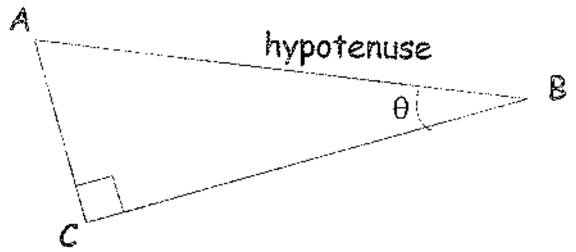
Basic Trig Ratios

Where Do I Use Trig?

The 3 basic trig. ratios are used only in right-angle triangles to find either a missing side or angle.

Don't forget to always use Pythagoras' Theorem when you know 2 sides in a right-angle triangle and you need to find the 3[rd] side.

There are 3 basic trig ratios, sine (sin), cosine (cos) and tangent (tan).
These represent the ratios of 2 named sides in a right-angle triangle.
The names of the sides depend on the position of the given (or required angle), so:

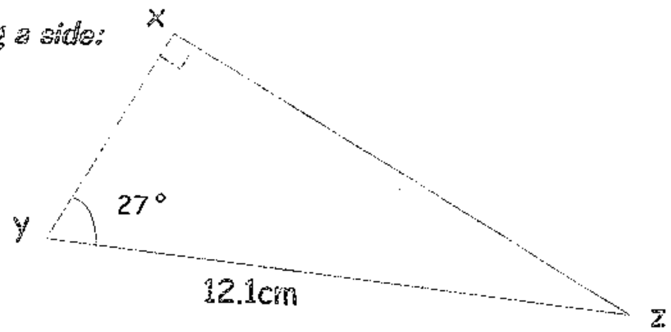


In this triangle, relative to θ, AB is the hypotenuse. (It's always opposite the right-angle). AC is the opposite side (because it's opposite to the given angle θ), which means that BC is the adjacent side because it's next to θ (and it's the only one left).

The ratios are:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Example 1 — Finding a side:



Firstly, identify the sides relative to 27° (∠ XYZ):

YZ is the hypotenuse
XY is the adjacent
XZ is the opposite

So we know the hypotenuse.
We need the opposite and so use the sine ratio:

$$\sin 27° = \frac{XZ}{12.1}$$

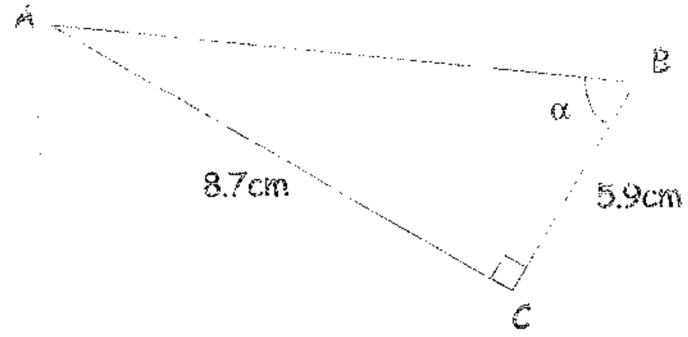
Multiply both sides by 12.1: $12.1 \times \sin 27° = XZ$

Use your calculator — make sure it's in D [or DEG] mode...

[1] [2] [.] [1] [x] [sin] [2] [7] [=]

$\boxed{XZ = 5.49\text{cm (2 d.p.)}}$

Example 2 — Finding an angle:



Relative to α, AC is the ***opposite*** *side*
BC is the ***adjacent*** *side*
so we use the tangent ratio:

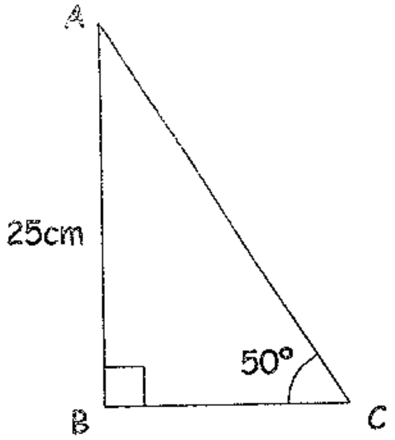
$$\tan \alpha = \frac{8.7}{5.9} = 1.474576271$$

$$\boxed{\alpha = 55.9^\circ \text{ (1 d.p.)}}$$

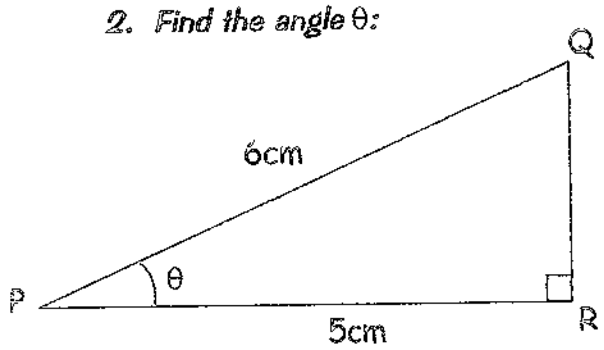
[SHIFT] [tan] [(] [8] [.] [7] [÷] [5] [.] [1] [1] [)] [=]

Now try these questions. Don't forget to give the answers to a sensible degree of accuracy.

1. Find the length of the hypotenuse AC:



2. Find the angle θ:



Solutions

1. AB is opposite. AC is hypotenuse.

$\sin 50^\circ = \frac{25}{AC}$

$AC = \frac{25}{\sin 50^\circ}$

$= 32.64$ cm (to 2 d.p.)

2. PQ is hypotenuse. PR is adjacent.

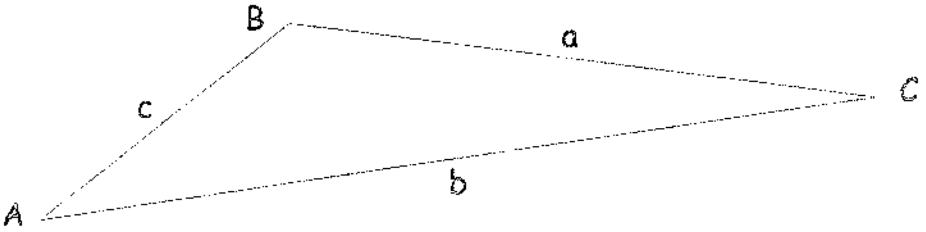
$\cos \theta = \frac{5}{6}$

$\theta = 33.6^\circ$ (to 1 d.p.)

The Sine Rule

Where Do I Use The Rules?

The sine and cosine rules are used in 'general triangles' (that don't contain a right-angle) to find missing lengths or angles.



The sine rule states that:

$$\frac{a}{\text{sin of angle } A} = \frac{b}{\text{sin of angle } B} \quad \text{or} \quad \frac{b}{\text{sin of angle } B} = \frac{c}{\text{sin of angle } C}$$

or even

$$\frac{a}{\text{sin of angle } A} = \frac{c}{\text{sin of angle } C}$$

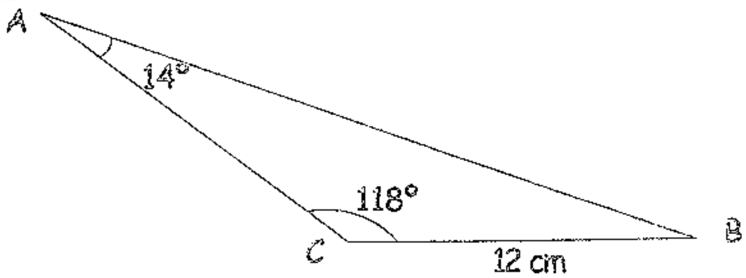
You get the picture...

When Do I Use The Sine Rule?

You can see from this rule that if you need to find the length of a side you must know the angle opposite that side as well as another side and its opposite angle.

Example:

Find the length of AB.



Another side — Angle opposite — $\dfrac{12}{\sin 14} = \dfrac{AB}{\sin 118}$ — length required — angle opposite required side

$$AB \times \sin 14 = 12 \times \sin 118$$

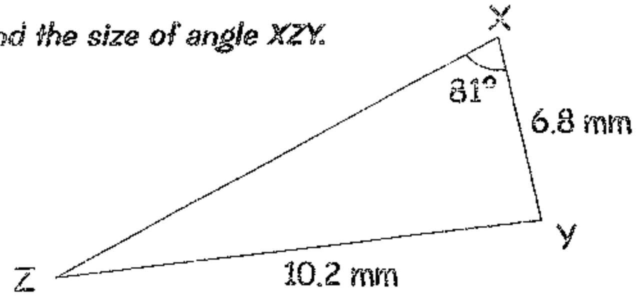
Divide through by sin 14: $AB = \dfrac{12 \times \sin 118}{\sin 14}$

$$\boxed{AB = 43.8 \text{ cm (to 1 d.p.)}}$$

(1 2 x sin 1 1 8) / sin 1 4 =

It's just as easy to find an angle...

Example: *Find the size of angle XZY.*



Sine rule: $\dfrac{6.8}{\sin \angle XZY} = \dfrac{10.2}{\sin 81}$

cross multiply: $10.2 \times \sin \angle XZY = 6.8 \times \sin 81$

divide both sides by 10.2: $\sin \angle XZY = \dfrac{6.8 \times \sin 81}{10.2}$

$= 0.658458893$

$\boxed{\angle XZY = 41.2° \text{ (1d.p.)}}$

Like looking for an angle in a haystack.

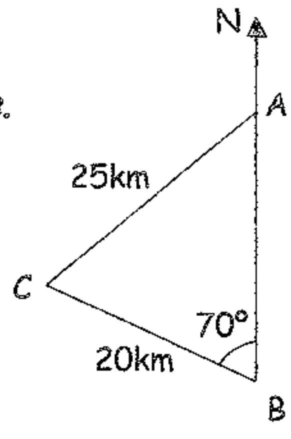
Now you try some:

1) *Two ships, A and B leave Mersey Docks (C) at the same time. Ship A travels at a bearing of 120° and ship B travels at a bearing of 100°. After 1 hour ship A has travelled 25 km and angle CBA is 130°.*

   Find the speed of ship B.

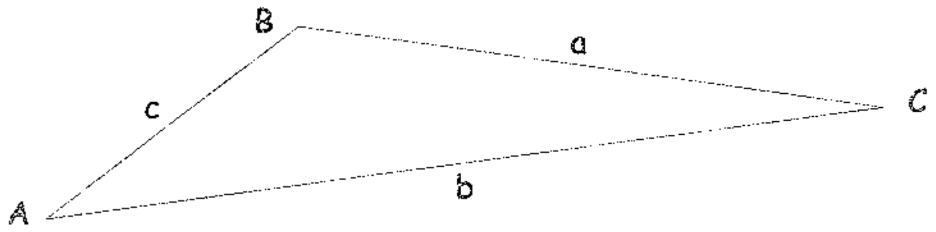
2) *Town A is due North of B. Town C is on a bearing of 290° from B and is 25 km from A and 20 km from B.*

   Find the bearing of C from A.



The Cosine Rule

Back to our original general triangle:



To find the length of a side we need to use the cosine rule in this form:

$$\boxed{a^2 = b^2 + c^2 - 2bc \times \cos A}$$

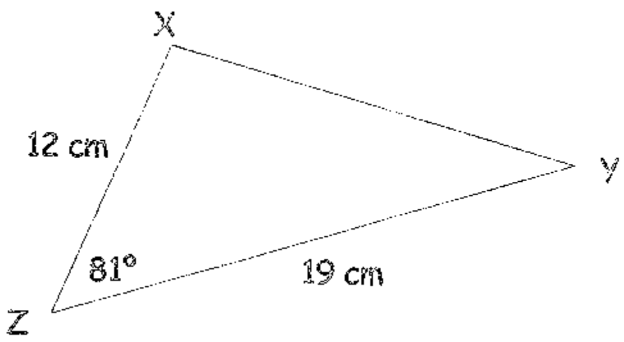
This formula can be rearranged as:

$$\boxed{\cos A = \frac{b^2 + c^2 - a^2}{2bc}}$$ *to find an angle.*

When Do I Use The Cosine Rule?

Firstly to find a side we need the lengths of two sides as well as the angle between the 2 known sides.

Example: *Find the length of XY.*



Using $a^2 = b^2 + c^2 - 2bc \times \cos A$

$XY^2 = 12^2 + 19^2 - (2 \times 12 \times 19 \times \cos 81)$

$= 433.6658839$

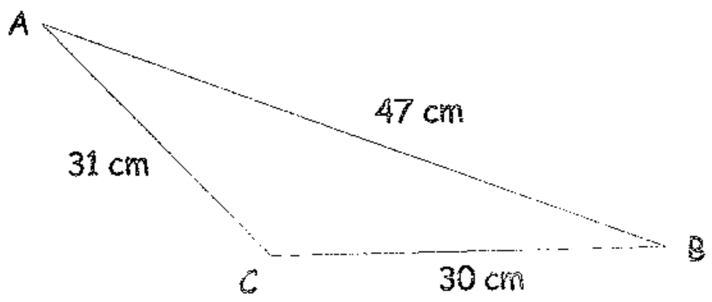
$XY = \sqrt{433.6658839}$

$\boxed{= 20.8\text{cm (1d.p.)}}$

The Cosine Rule

And to find an angle...

Example: *Find angle ACB.*



$$\cos \angle ACB = \frac{30^2 + 31^2 - 47^2}{2 \times 30 \times 31}$$

$$\cos \angle ACB = -0.187096774$$

$$\boxed{\angle ACB = 100.8° \text{ (1 d.p.)}}$$

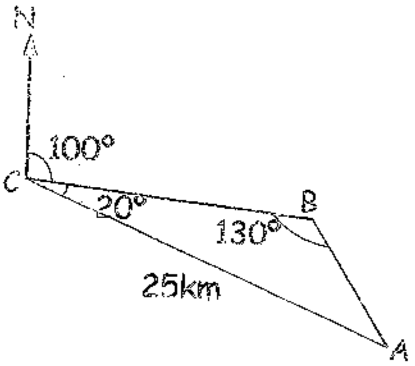
Now your turn...

1) Use the cosine rule to find the length of the third side of the triangle with sides AB = 5cm, AC = 7cm and BAC = 35 .

2) Use the cosine rule to find ABC to 0.1 in the triangle which has sides: AB = 14.2cm, BC = 9.3cm and AC = 11.6cm.

Solutions

1) First draw a diagram:



$\angle A = 180 - 130 - 20$
$= 30°$

From the sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 30} = \frac{25}{\sin 130}$$

$$a = \frac{25}{\sin 130} \times \sin 30$$

$a = 16.3$ km (1dp)

So, the speed of ship B is 16.3 km/hr.

2) ***Because the bearing of C from A is 290°, $\angle ABC$ is 70°***

From the sine rule:

$$\frac{\sin A}{a} = \frac{\sin B}{b} \qquad \frac{\sin A}{20} = \frac{\sin 70}{25}$$

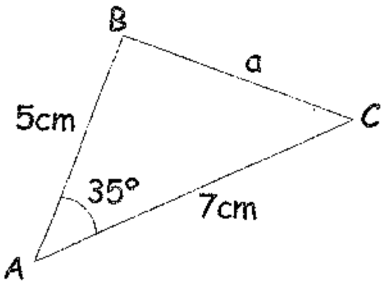
$$\sin A = \frac{\sin 70}{25} \times 20 = 0.7517....$$

$$A = 48.7° \text{ to 1dp.}$$

So, the bearing of C from A is 180 + 48.7 = 228.7° (to 1 d.p.)

Solutions to Cosine Rule Questions:

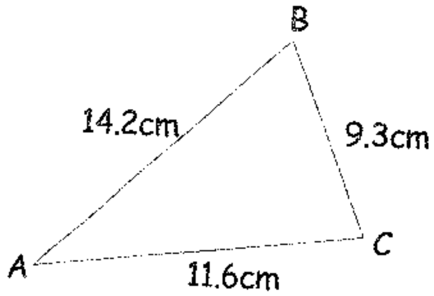
1)



By the cosine rule:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 7^2 + 5^2 - (2 \times 7 \times 5)\cos 35$$
$$= 74 - 70 \times 0.8192$$
$$= 74 - 57.34$$
$$a^2 = 16.66$$
$$a = \sqrt{16.66} = 4.08\text{cm} \quad (3 \text{ s.f.})$$

2)



By the cosine rule:

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{9.3^2 + 14.2^2 - 11.6^2}{2 \times 9.3 \times 14.2}$$

$$= \frac{153.57}{264.12} = 0.58144$$

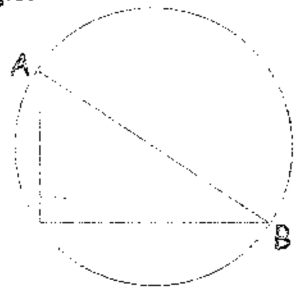
$$\angle ABC = 54.4° \text{ (to 0.1°)}$$

Circle Properties

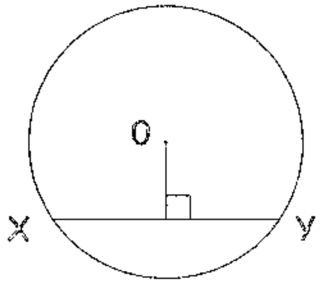
As well as knowing the names of all the bits in a circle, there are some important facts that you must also know.

Remember — a tangent is a straight line that touches a circle at one point only.

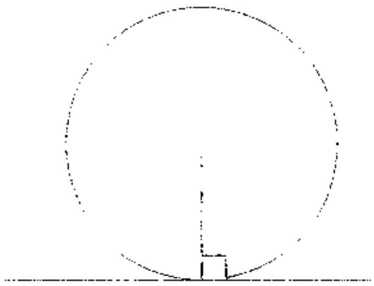
If AB is the diameter of a circle, any angle drawn at the circumference on this diameter is always a right angle.



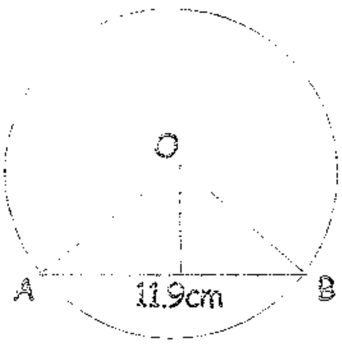
A perpendicular from the centre of a circle O to a chord bisects that chord.



The angle between a radius and the point of tangent is a right angle.



Example: **O is the centre of the circle radius 6.8cm. If AB is a chord of 11.9 cm, find the area of the triangle.**



OK. To find the area first we need the perpendicular height, and that's the line that's going to cut AB in half, so it's over to Pythagoras again:

$h^2 = 6.8^2 - 5.95^2$

That makes the perpendicular height $h = 3.292035844$ cm

So now the area's easy — just half of 11.9 multiplied by the height we've just found.

In other words: Area = 19.6cm (to 1 d.p)

Now your turn... **Points A, B, C and D lie in clockwise order on a circle. AB = 4cm, BC = 3cm and CD = 3.5cm. AC is a diameter. Find the area of the quadrilateral ABCD.**

<u>Solution:</u>

Angles ADC and ABC must both be right angles if AC is a diameter.
We can see straight away that AC = 5cm (ABC is a 3,4,5 Δ)

The area of ΔABC is just $\frac{3 \times 4}{2} = 6\text{cm}^2$

Now we need the length of AD so it's you-know-who again:

$$AD = \sqrt{(5^2 - 3.5^2)}$$
$$= 3.570714214 \text{ cm}$$

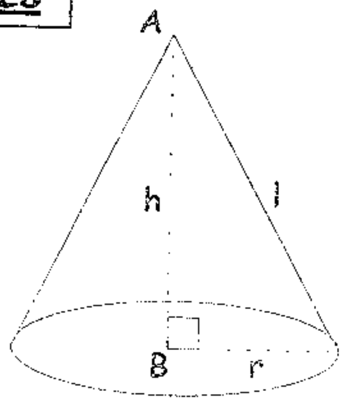
So the area of ΔADC is going to be $\frac{3.5 \times 3.570714214}{2} = 6.248749875\text{cm}^2$

Adding the two areas together and rounding that comes to a grand total of <u>12.2cm²</u> *(to 1 d.p)*

Cones, Spheres and Circles

It's useful to remember some important facts about cones and spheres.

Cones



In this cone, where A is directly above B,

- the area of the circular base is πr^2
- the curved surface area is $\pi r l$

That means that the total surface must be $\pi r^2 + \pi r l$ *— or* $\pi r(r + l)$ *if you're being really smart.*

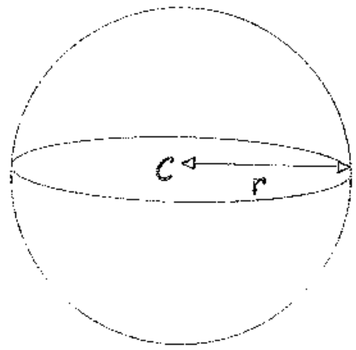
- The volume of the cone is $\dfrac{\pi r^2 \times h}{3}$

Spheres

In this sphere, where C is the centre and r the radius, the surface area is calculated by using:

Surface area $= 4\pi r^2$

And the volume is $\dfrac{4\pi r^3}{3}$



Example: A spinning top consists of a hemisphere sitting on top of a cone.
The diameter of the sphere is 9cm and the perpendicular height of the cone is 14.5cm.
Find the total surface area and the volume of the shape.

Total surface area is $\pi r l + \dfrac{4\pi r^2}{2}$ (Don't forget we only need the area of the curved surface of the cone and only half the surface area of the sphere)

We'll need the slant height to be able to use this formula, so its back to good old Pythagoras to help out again:

Slant height^2 = perpendicular height^2 + radius^2

Slant height $= \sqrt{(14.5^2 + 4.5^2)} = \underline{15.18222645}$ cm

Back to the surface area:

Surface area $= (\pi \times 4.5 \times 15.18222645) + \dfrac{(4 \times \pi \times 4.5^2)}{2}$

$= 341.8681723 \qquad = \underline{341.9 \text{ cm}^2}$ to 1 d.p.

Now for the total volume:

We want the volume of the cone + half the volume of the sphere

Total volume $= \dfrac{(\pi \times 4.5^2 \times 14.5)}{3} + \dfrac{(4 \times \pi \times 4.5^3)}{6}$

$= \underline{498.3 \text{ cm}^3}$ (to 1 d.p)

Questions on Cones, Spheres and Circles

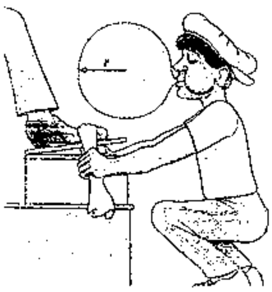
Here's a couple for you to try:

1) *A cone has a circular base of diameter 11.2cm and a slant height of 14.3cm. Find its volume.*

2) *The surface area of a sphere is 3256 cm^2. Calculate its radius.*

Solutions:

(You might want to draw a diagram first, it can really help)



1) *Firstly, the perpendicular height, by Pythagoras.*

$$\text{Height} = \sqrt{(14.3^2 - 5.6^2)}$$
$$= 13.15788737 \text{ cm}$$

and then the Volume
$$= \frac{\pi \times 5.6^2 \times 13.15788737}{3}$$
$$= 432.1 \text{ cm}^3 \text{ (to 1 d.p)}$$

2) *If the surface area of a sphere is 3256 cm^2*

Then $4 \times \pi \times r^2 = 3256$

So $r^2 = \dfrac{3256}{4\pi}$

$= 259.1042474$

So the radius is $\underline{16.1\text{ cm}}$ *(1 d.p)*

Proportion

Two quantities are in direct proportion if one is a constant multiple of the other.
An example of direct proportion could be:
The number of books sold and the total cost of them.

> You can show proportionality using algebra.
> If x is proportional to y we can write this as: $x \propto y$

Proportionality Equations

This proportionality statement can be replaced by a proportionality equation: $x = ky$
where k is the constant of proportionality or the constant multiple.
Here's how you work out what k is:

If $x \propto y$ then $x = ky$

If we know that $x = 18$ when $y = 6$ then we can see that $k = 3$ in this case.

So: $x = 3y$

Squared and cubed terms

We can have direct proportion that involves squared and cubed terms.
If a is proportional to the square of b then:

$$a \propto b^2 \quad \text{and} \quad a = kb^2$$

If we know that $a = 24$ when $b = 2$ then: $24 = 4k$

so $k = 6$ ie $\boxed{a = 6b^2}$

Inverse proportion

Two quantities are in inverse proportion if one quantity is multiplied by a given number and the other is divided by the same number.

You write a is inversely proportional to b as: $a \propto \frac{1}{b}$

> e.g p is inversely proportional to q and $p = 24$ when $q = 2$.
> 1. Find the proportionality equation for p and q.
> 2. Find the value of p when $q = 4$

1. $p \propto \frac{1}{q}$ so $p = \frac{k}{q}$

$p = 24$ when $q = 2$

$24 = \frac{k}{2}$ so $k = 48$

So the proportionality equation is $\boxed{p = \frac{48}{q}}$

2. When $q = 4$ then $p = \frac{48}{q}$

$p = \frac{48}{4}$ so $\boxed{p = 12}$

Proportion

Now you have a go at some:

1. *Given that y is directly proportional to x^3 and that $y = 9$ when $x = 3$, find y in terms of x.*
2. *Given that p is inversely proportional to $\sqrt{q}$ and that $p = 2$ when $q = 25$; find p when $q = 36$*

Solutions

1. $y \propto x^3$ **so** $y = kx^3$

   $y = 9$ *when* $x = 3$ **so** $9 = 27k$

   $k = \frac{1}{3}$ **and** $y = \frac{1}{3}x^3$

2. $p \propto \frac{1}{\sqrt{q}}$ **so** $p = \frac{k}{\sqrt{q}}$

   when $p=2$ *then* $q=25$ *so:*

   $2 = \frac{k}{\sqrt{25}}$ **ie** $k = 10$

   $p = \frac{10}{\sqrt{q}}$

   so when $q=36$, $p = \frac{10}{6} = \frac{5}{3}$

Index

A

adding/subtracting fractions 4
algebraic fractions 3, 12-14

B

bases 6, 8

C

chords 40
circles 40-42
circumference 40
completing the square 18
cones 41, 42
cosine rule 37, 38

D

decimals 1, 2
denominators 3
diameters 40, 41
difference of two squares 21
direct proportion 43, 44
distances between points 30-32
dividing fractions 4
dividing indices 7

E

elimination method 23-25
equations of straight lines 29

F

fractional indices 6
fractions 1, 3-5

G

gradients 26, 27, 29

I

indices 6-10
integers 1, 2
intersections 17
inverse proportion 43, 44
irrational numbers 1, 2

L

linear simultaneous equations 23-25
lowest common multiples 3

N

negative indices 7
non-repeating decimals 1, 2

P

perpendiculars 40, 41
powers 6-10
proportion 43-44
Pythagoras' Theorem 30, 33, 40

Q

quadratic equations 17-22
quadratic formula 19

R

radius 41
rational numbers 1, 2
real numbers 1, 2
recurring decimals 1, 2
right-angle triangles 30, 33, 34
roots 1, 2, 6

S

simplifying 8, 9, 12, 13
sine rule 35, 36
spheres 41, 42
straight line graphs 26-32
substitution method 24, 25
surds 1, 2, 6
surface area 41, 42

T

tangents 40
top-heavy fractions 3
trigonometry 33-42

V

volume 41

W

whole numbers 1, 2

Y

IBR51 0602-486

GCSE to A level Transition Work

Expanding brackets and simplifying expressions	2
Surds and rationalising the denominator	5
Rules of Indices	10
Factorising Expressions	15
Completing the Square	19
Solving Quadratic Equations	22
Sketching Quadratic Graphs	29
Solving linear simultaneous equations	33
Solving quadratics and linear simultaneous equations	38
Solving simultaneous equations graphically	41
Linear inequalities	45
Quadratic inequalities	48
Sketching cubic and reciprocal graphs	51
Translating graphs	56
Straight-line graphs	66
Parallel and perpendicular graphs	70
Pythagoras' theorem	74
Proportion	78
Circle theorems	84
Trigonometry	93
Rearranging equations	107
Volume and surface area of 3D shapes	110
Area under a graph	115

Expanding brackets and simplifying expressions

A LEVEL LINKS **Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
- When you expand two linear expressions, each with two terms of the form $ax + b$, where $a \neq 0$ and $b \neq 0$, you create four terms. Two of these can usually be simplified by collecting like terms.

Examples

Example 1 Expand $4(3x - 2)$

$4(3x - 2) = 12x - 8$	Multiply everything inside the bracket by the 4 outside the bracket

Example 2 Expand and simplify $3(x + 5) - 4(2x + 3)$

$3(x + 5) - 4(2x + 3)$ $= 3x + 15 - 8x - 12$	**1** Expand each set of brackets separately by multiplying $(x + 5)$ by 3 and $(2x + 3)$ by -4
$= 3 - 5x$	**2** Simplify by collecting like terms: $3x - 8x = -5x$ and $15 - 12 = 3$

Example 3 Expand and simplify $(x + 3)(x + 2)$

$(x + 3)(x + 2)$ $= x(x + 2) + 3(x + 2)$ $= x^2 + 2x + 3x + 6$	**1** Expand the brackets by multiplying $(x + 2)$ by x and $(x + 2)$ by 3
$= x^2 + 5x + 6$	**2** Simplify by collecting like terms: $2x + 3x = 5x$

Example 4 Expand and simplify $(x - 5)(2x + 3)$

$(x - 5)(2x + 3)$ $= x(2x + 3) - 5(2x + 3)$ $= 2x^2 + 3x - 10x - 15$	**1** Expand the brackets by multiplying $(2x + 3)$ by x and $(2x + 3)$ by -5
$= 2x^2 - 7x - 15$	**2** Simplify by collecting like terms: $3x - 10x = -7x$

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Practice

> **Watch out!**
>
> When multiplying (or dividing) positive and negative numbers, if the signs are the same the answer is ‘+’; if the signs are different the answer is ‘–’.

1 Expand.

a $3(2x - 1)$ **b** $-2(5pq + 4q^2)$

c $-(3xy - 2y^2)$

2 Expand and simplify.

a $7(3x + 5) + 6(2x - 8)$ **b** $8(5p - 2) - 3(4p + 9)$

c $9(3s + 1) - 5(6s - 10)$ **d** $2(4x - 3) - (3x + 5)$

3 Expand.

a $3x(4x + 8)$ **b** $4k(5k^2 - 12)$

c $-2h(6h^2 + 11h - 5)$ **d** $-3s(4s^2 - 7s + 2)$

4 Expand and simplify.

a $3(y^2 - 8) - 4(y^2 - 5)$ **b** $2x(x + 5) + 3x(x - 7)$

c $4p(2p - 1) - 3p(5p - 2)$ **d** $3b(4b - 3) - b(6b - 9)$

5 Expand $\frac{1}{2}(2y - 8)$

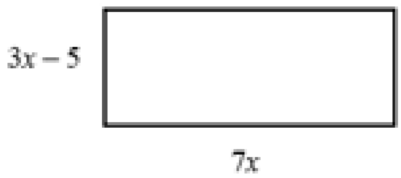
6 Expand and simplify.

a $13 - 2(m + 7)$ **b** $5p(p^2 + 6p) - 9p(2p - 3)$

7 The diagram shows a rectangle.

Write down an expression, in terms of x, for the area of the rectangle.

Show that the area of the rectangle can be written as $21x^2 - 35x$



8 Expand and simplify.

a $(x + 4)(x + 5)$ **b** $(x + 7)(x + 3)$

c $(x + 7)(x - 2)$ **d** $(x + 5)(x - 5)$

e $(2x + 3)(x - 1)$ **f** $(3x - 2)(2x + 1)$

g $(5x - 3)(2x - 5)$ **h** $(3x - 2)(7 + 4x)$

i $(3x + 4y)(5y + 6x)$ **j** $(x + 5)^2$

k $(2x - 7)^2$ **l** $(4x - 3y)^2$

Extend

9 Expand and simplify $(x + 3)^2 + (x - 4)^2$

10 Expand and simplify.

a $\left(x + \frac{1}{x}\right)\left(x - \frac{2}{x}\right)$ **b** $\left(x + \frac{1}{x}\right)^2$

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Answers

1 **a** $6x - 3$ **b** $-10pq - 8q^2$
c $-3xy + 2y^2$

2 **a** $21x + 35 + 12x - 48 = 33x - 13$
b $40p - 16 - 12p - 27 = 28p - 43$
c $27s + 9 - 30s + 50 = -3s + 59 = 59 - 3s$
d $8x - 6 - 3x - 5 = 5x - 11$

3 **a** $12x^2 + 24x$ **b** $20k^3 - 48k$
c $10h - 12h^3 - 22h^2$ **d** $21s^2 - 21s^3 - 6s$

4 **a** $-y^2 - 4$ **b** $5x^2 - 11x$
c $2p - 7p^2$ **d** $6b^2$

5 $y - 4$

6 **a** $-1 - 2m$ **b** $5p^3 + 12p^2 + 27p$

7 $7x(3x - 5) = 21x^2 - 35x$

8 **a** $x^2 + 9x + 20$ **b** $x^2 + 10x + 21$
c $x^2 + 5x - 14$ **d** $x^2 - 25$
e $2x^2 + x - 3$ **f** $6x^2 - x - 2$
g $10x^2 - 31x + 15$ **h** $12x^2 + 13x - 14$
i $18x^2 + 39xy + 20y^2$ **j** $x^2 + 10x + 25$
k $4x^2 - 28x + 49$ **l** $16x^2 - 24xy + 9y^2$

9 $2x^2 - 2x + 25$

10 **a** $x^2 - 1 - \dfrac{2}{x^2}$ **b** $x^2 + 2 + \dfrac{1}{x^2}$

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Surds and rationalising the denominator

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}, \sqrt{3}, \sqrt{5}$, etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\dfrac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd $\sqrt{b}$
- To rationalise $\dfrac{a}{b+\sqrt{c}}$ you multiply the numerator and denominator by $b-\sqrt{c}$

Examples

Example 1 Simplify $\sqrt{50}$

$\sqrt{50} = \sqrt{25 \times 2}$	**1** Choose two numbers that are factors of 50. One of the factors must be a square number
$= \sqrt{25} \times \sqrt{2}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 5 \times \sqrt{2}$	**3** Use $\sqrt{25} = 5$
$= 5\sqrt{2}$	

Example 2 Simplify $\sqrt{147} - 2\sqrt{12}$

$\sqrt{147} - 2\sqrt{12}$ $= \sqrt{49 \times 3} - 2\sqrt{4 \times 3}$	**1** Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number
$= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3}$	**3** Use $\sqrt{49} = 7$ and $\sqrt{4} = 2$
$= 7\sqrt{3} - 4\sqrt{3}$ $= 3\sqrt{3}$	**4** Collect like terms

Example 3 Simplify $\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$

$\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$ $= \sqrt{49}-\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}-\sqrt{4}$ $= 7 - 2$ $= 5$	**1** Expand the brackets. A common mistake here is to write $\left(\sqrt{7}\right)^2 = 49$ **2** Collect like terms: $-\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}$ $= -\sqrt{7}\sqrt{2}+\sqrt{7}\sqrt{2} = 0$

Example 4 Rationalise $\frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ $= \frac{1\times\sqrt{3}}{\sqrt{9}}$ $= \frac{\sqrt{3}}{3}$	**1** Multiply the numerator and denominator by $\sqrt{3}$ **2** Use $\sqrt{9}=3$

Example 5 Rationalise and simplify $\frac{\sqrt{2}}{\sqrt{12}}$

$\frac{\sqrt{2}}{\sqrt{12}} = \frac{\sqrt{2}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}}$ $= \frac{\sqrt{2}\times\sqrt{4\times3}}{12}$ $= \frac{2\sqrt{2}\sqrt{3}}{12}$ $= \frac{\sqrt{2}\sqrt{3}}{6}$	**1** Multiply the numerator and denominator by $\sqrt{12}$ **2** Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number **3** Use the rule $\sqrt{ab}=\sqrt{a}\times\sqrt{b}$ **4** Use $\sqrt{4}=2$ **5** Simplify the fraction: $\frac{2}{12}$ simplifies to $\frac{1}{6}$

Example 6 Rationalise and simplify $\dfrac{3}{2+\sqrt{5}}$

$\dfrac{3}{2+\sqrt{5}} = \dfrac{3}{2+\sqrt{5}} \times \dfrac{2-\sqrt{5}}{2-\sqrt{5}}$	**1** Multiply the numerator and denominator by $2-\sqrt{5}$
$= \dfrac{3\left(2-\sqrt{5}\right)}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}$	**2** Expand the brackets
$= \dfrac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$	**3** Simplify the fraction
$= \dfrac{6-3\sqrt{5}}{-1}$	**4** Divide the numerator by −1 Remember to change the sign of all terms when dividing by −1
$= 3\sqrt{5}-6$	

Practice

1 Simplify.

a $\sqrt{45}$ **b** $\sqrt{125}$

c $\sqrt{48}$ **d** $\sqrt{175}$

e $\sqrt{300}$ **f** $\sqrt{28}$

g $\sqrt{72}$ **h** $\sqrt{162}$

Hint

One of the two numbers you choose at the start must be a square number.

2 Simplify.

a $\sqrt{72}+\sqrt{162}$ **b** $\sqrt{45}-2\sqrt{5}$

c $\sqrt{50}-\sqrt{8}$ **d** $\sqrt{75}-\sqrt{48}$

e $2\sqrt{28}+\sqrt{28}$ **f** $2\sqrt{12}-\sqrt{12}+\sqrt{27}$

Watch out!

Check you have chosen the highest square number at the start.

3 Expand and simplify.

a $(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$ **b** $(3+\sqrt{3})(5-\sqrt{12})$

c $(4-\sqrt{5})(\sqrt{45}+2)$ **d** $(5+\sqrt{2})(6-\sqrt{8})$

4 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{5}}$ **b** $\frac{1}{\sqrt{11}}$

c $\frac{2}{\sqrt{7}}$ **d** $\frac{2}{\sqrt{8}}$

e $\frac{2}{\sqrt{2}}$ **f** $\frac{5}{\sqrt{5}}$

g $\frac{\sqrt{8}}{\sqrt{24}}$ **h** $\frac{\sqrt{5}}{\sqrt{45}}$

5 Rationalise and simplify.

a $\frac{1}{3-\sqrt{5}}$ **b** $\frac{2}{4+\sqrt{3}}$ **c** $\frac{6}{5-\sqrt{2}}$

Extend

6 Expand and simplify $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)$

7 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{9}-\sqrt{8}}$ **b** $\frac{1}{\sqrt{x}-\sqrt{y}}$

Answers

1 **a** $3\sqrt{5}$ **b** $5\sqrt{5}$

c $4\sqrt{3}$ **d** $5\sqrt{7}$

e $10\sqrt{3}$ **f** $2\sqrt{7}$

g $6\sqrt{2}$ **h** $9\sqrt{2}$

2 **a** $15\sqrt{2}$ **b** $\sqrt{5}$

c $3\sqrt{2}$ **d** $\sqrt{3}$

e $6\sqrt{7}$ **f** $5\sqrt{3}$

3 **a** -1 **b** $9-\sqrt{3}$

c $10\sqrt{5}-7$ **d** $26-4\sqrt{2}$

4 **a** $\frac{\sqrt{5}}{5}$ **b** $\frac{\sqrt{11}}{11}$

c $\frac{2\sqrt{7}}{7}$ **d** $\frac{\sqrt{2}}{2}$

e $\sqrt{2}$ **f** $\sqrt{5}$

g $\frac{\sqrt{3}}{3}$ **h** $\frac{1}{3}$

5 **a** $\frac{3+\sqrt{5}}{4}$ **b** $\frac{2(4-\sqrt{3})}{13}$ **c** $\frac{6(5+\sqrt{2})}{23}$

6 $x-y$

7 **a** $3+2\sqrt{2}$ **b** $\frac{\sqrt{x}+\sqrt{y}}{x-y}$

Rules of indices

A LEVEL LINKS **Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- $a^m \times a^n = a^{m+n}$
- $\dfrac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the nth root of a
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $a^{-m} = \dfrac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Examples

Example 1 Evaluate 10^0

$10^0 = 1$	Any value raised to the power of zero is equal to 1

Example 2 Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$

Example 3 Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$ $= 3^2$ $= 9$	**1** Use the rule $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ **2** Use $\sqrt[3]{27} = 3$

Example 4 Evaluate 4^{-2}

$4^{-2} = \frac{1}{4^2}$ $= \frac{1}{16}$	**1** Use the rule $a^{-m} = \frac{1}{a^m}$ **2** Use $4^2 = 16$

Example 5 Simplify $\frac{6x^5}{2x^2}$

$\frac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$

Example 6 Simplify $\frac{x^3 \times x^5}{x^4}$

$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$ $= x^{8-4} = x^4$	**1** Use the rule $a^m \times a^n = a^{m+n}$ **2** Use the rule $\frac{a^m}{a^n} = a^{m-n}$

Example 7 Write $\frac{1}{3x}$ as a single power of x

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged

Example 8 Write $\frac{4}{\sqrt{x}}$ as a single power of x

$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$ $= 4x^{-\frac{1}{2}}$	**1** Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ **2** Use the rule $\frac{1}{a^m} = a^{-m}$

Practice

1 Evaluate.

a 14^0 **b** 3^0 **c** 5^0 **d** x^0

2 Evaluate.

a $49^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$ **c** $125^{\frac{1}{3}}$ **d** $16^{\frac{1}{4}}$

3 Evaluate.

a $25^{\frac{3}{2}}$ **b** $8^{\frac{5}{3}}$ **c** $49^{\frac{3}{2}}$ **d** $16^{\frac{3}{4}}$

4 Evaluate.

a 5^{-2} **b** 4^{-3} **c** 2^{-5} **d** 6^{-2}

5 Simplify.

a $\dfrac{3x^2 \times x^3}{2x^2}$ **b** $\dfrac{10x^5}{2x^2 \times x}$

c $\dfrac{3x \times 2x^3}{2x^3}$ **d** $\dfrac{7x^3y^2}{14x^5y}$

e $\dfrac{y^2}{y^{\frac{1}{2}} \times y}$ **f** $\dfrac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$

g $\dfrac{(2x^2)^3}{4x^0}$ **h** $\dfrac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

> **Watch out!**
>
> Remember that any value raised to the power of zero is 1. This is the rule $a^0 = 1$.

6 Evaluate.

a $4^{-\frac{1}{2}}$ **b** $27^{-\frac{2}{3}}$ **c** $9^{-\frac{1}{2}} \times 2^3$

d $16^{\frac{1}{4}} \times 2^{-3}$ **e** $\left(\dfrac{9}{16}\right)^{-\frac{1}{2}}$ **f** $\left(\dfrac{27}{64}\right)^{-\frac{2}{3}}$

7 Write the following as a single power of x.

a $\dfrac{1}{x}$ **b** $\dfrac{1}{x^7}$ **c** $\sqrt[4]{x}$

d $\sqrt[5]{x^2}$ **e** $\dfrac{1}{\sqrt[3]{x}}$ **f** $\dfrac{1}{\sqrt[3]{x^2}}$

8 Write the following without negative or fractional powers.

a x^{-3} **b** x^0 **c** $x^{\frac{1}{5}}$

d $x^{\frac{2}{5}}$ **e** $x^{-\frac{1}{2}}$ **f** $x^{-\frac{3}{4}}$

9 Write the following in the form ax^n.

a $5\sqrt{x}$ **b** $\dfrac{2}{x^3}$ **c** $\dfrac{1}{3x^4}$

d $\dfrac{2}{\sqrt{x}}$ **e** $\dfrac{4}{\sqrt[3]{x}}$ **f** 3

Extend

10 Write as sums of powers of x.

a $\dfrac{x^5+1}{x^2}$ **b** $x^2\left(x+\dfrac{1}{x}\right)$ **c** $x^{-4}\left(x^2+\dfrac{1}{x^3}\right)$

Answers

1	**a**	1	**b**	1	**c**	1	**d**	1
2	**a**	7	**b**	4	**c**	5	**d**	2
3	**a**	125	**b**	32	**c**	343	**d**	8
4	**a**	$\frac{1}{25}$	**b**	$\frac{1}{64}$	**c**	$\frac{1}{32}$	**d**	$\frac{1}{36}$
5	**a**	$\frac{3x^3}{2}$	**b**	$5x^2$				
	c	$3x$	**d**	$\frac{y}{2x^2}$				
	e	$y^{\frac{1}{2}}$	**f**	c^{-3}				
	g	$2x^6$	**h**	x				
6	**a**	$\frac{1}{2}$	**b**	$\frac{1}{9}$	**c**	$\frac{8}{3}$		
	d	$\frac{1}{4}$	**e**	$\frac{4}{3}$	**f**	$\frac{16}{9}$		
7	**a**	x^{-1}	**b**	x^{-7}	**c**	$x^{\frac{1}{4}}$		
	d	$x^{\frac{2}{5}}$	**e**	$x^{-\frac{1}{3}}$	**f**	$x^{-\frac{2}{3}}$		
8	**a**	$\frac{1}{x^3}$	**b**	1	**c**	$\sqrt[5]{x}$		
	d	$\sqrt[5]{x^2}$	**e**	$\frac{1}{\sqrt{x}}$	**f**	$\frac{1}{\sqrt[4]{x^3}}$		
9	**a**	$5x^{\frac{1}{2}}$	**b**	$2x^{-3}$	**c**	$\frac{1}{3}x^{-4}$		
	d	$2x^{-\frac{1}{2}}$	**e**	$4x^{-\frac{1}{3}}$	**f**	$3x^0$		
10	**a**	$x^3 + x^{-2}$	**b**	$x^3 + x$	**c**	$x^{-2} + x^{-7}$		

Factorising expressions

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose product is ac.
- An expression in the form $x^2 – y^2$ is called the difference of two squares. It factorises to $(x – y)(x + y)$.

Examples

Example 1 Factorise $15x^2y^3 + 9x^4y$

$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$	The highest common factor is $3x^2y$. So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets

Example 2 Factorise $4x^2 – 25y^2$

$4x^2 – 25y^2 = (2x + 5y)(2x − 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

Example 3 Factorise $x^2 + 3x – 10$

$b = 3$, $ac = −10$	**1** Work out the two factors of $ac = −10$ which add to give $b = 3$ (5 and −2)
So $x^2 + 3x – 10 = x^2 + 5x – 2x – 10$	**2** Rewrite the b term ($3x$) using these two factors
$= x(x + 5) – 2(x + 5)$	**3** Factorise the first two terms and the last two terms
$= (x + 5)(x – 2)$	**4** $(x + 5)$ is a factor of both terms

Example 4 Factorise $6x^2 - 11x - 10$

$b = -11$, $ac = -60$ So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$	**1** Work out the two factors of $ac = -60$ which add to give $b = -11$ (-15 and 4) **2** Rewrite the b term ($-11x$) using these two factors **3** Factorise the first two terms and the last two terms **4** $(2x - 5)$ is a factor of both terms

Example 5 Simplify $\dfrac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

$\dfrac{x^2 - 4x - 21}{2x^2 + 9x + 9}$	**1** Factorise the numerator and the denominator
For the numerator: $b = -4$, $ac = -21$	**2** Work out the two factors of $ac = -21$ which add to give $b = -4$ (-7 and 3)
So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$ $= x(x - 7) + 3(x - 7)$ $= (x - 7)(x + 3)$	**3** Rewrite the b term ($-4x$) using these two factors **4** Factorise the first two terms and the last two terms **5** $(x - 7)$ is a factor of both terms
For the denominator: $b = 9$, $ac = 18$	**6** Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3)
So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$ $= 2x(x + 3) + 3(x + 3)$ $= (x + 3)(2x + 3)$	**7** Rewrite the b term ($9x$) using these two factors **8** Factorise the first two terms and the last two terms **9** $(x + 3)$ is a factor of both terms
So $\dfrac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \dfrac{(x-7)(x+3)}{(x+3)(2x+3)}$ $= \dfrac{x-7}{2x+3}$	**10** $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1

Practice

1 Factorise.

a $6x^4y^3 - 10x^3y^4$ **b** $21a^3b^5 + 35a^5b^2$

c $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

> **Hint**
>
> Take the highest common factor outside the bracket.

2 Factorise

a $x^2 + 7x + 12$ **b** $x^2 + 5x - 14$

c $x^2 - 11x + 30$ **d** $x^2 - 5x - 24$

e $x^2 - 7x - 18$ **f** $x^2 + x - 20$

g $x^2 - 3x - 40$ **h** $x^2 + 3x - 28$

3 Factorise

a $36x^2 - 49y^2$ **b** $4x^2 - 81y^2$

c $18a^2 - 200b^2c^2$

4 Factorise

a $2x^2 + x - 3$ **b** $6x^2 + 17x + 5$

c $2x^2 + 7x + 3$ **d** $9x^2 - 15x + 4$

e $10x^2 + 21x + 9$ **f** $12x^2 - 38x + 20$

5 Simplify the algebraic fractions.

a $\dfrac{2x^2+4x}{x^2-x}$ **b** $\dfrac{x^2+3x}{x^2+2x-3}$

c $\dfrac{x^2-2x-8}{x^2-4x}$ **d** $\dfrac{x^2-5x}{x^2-25}$

e $\dfrac{x^2-x-12}{x^2-4x}$ **f** $\dfrac{2x^2+14x}{2x^2+4x-70}$

6 Simplify

a $\dfrac{9x^2-16}{3x^2+17x-28}$ **b** $\dfrac{2x^2-7x-15}{3x^2-17x+10}$

c $\dfrac{4-25x^2}{10x^2-11x-6}$ **d** $\dfrac{6x^2-x-1}{2x^2+7x-4}$

Extend

7 Simplify $\sqrt{x^2+10x+25}$

8 Simplify $\dfrac{(x+2)^2+3(x+2)^2}{x^2-4}$

Answers

1 **a** $2x^3y^3(3x - 5y)$ **b** $7a^3b^2(3b^3 + 5a^2)$

c $5x^2y^2(5 - 2x + 3y)$

2 **a** $(x + 3)(x + 4)$ **b** $(x + 7)(x - 2)$

c $(x - 5)(x - 6)$ **d** $(x - 8)(x + 3)$

e $(x - 9)(x + 2)$ **f** $(x + 5)(x - 4)$

g $(x - 8)(x + 5)$ **h** $(x + 7)(x - 4)$

3 **a** $(6x - 7y)(6x + 7y)$ **b** $(2x - 9y)(2x + 9y)$

c $2(3a - 10bc)(3a + 10bc)$

4 **a** $(x - 1)(2x + 3)$ **b** $(3x + 1)(2x + 5)$

c $(2x + 1)(x + 3)$ **d** $(3x - 1)(3x - 4)$

e $(5x + 3)(2x + 3)$ **f** $2(3x - 2)(2x - 5)$

5 **a** $\frac{2(x+2)}{x-1}$ **b** $\frac{x}{x-1}$

c $\frac{x+2}{x}$ **d** $\frac{x}{x+5}$

e $\frac{x+3}{x}$ **f** $\frac{x}{x-5}$

6 **a** $\frac{3x+4}{x+7}$ **b** $\frac{2x+3}{3x-2}$

c $\frac{2-5x}{2x-3}$ **d** $\frac{3x+1}{x+4}$

7 $(x + 5)$

8 $\frac{4(x+2)}{x-2}$

Completing the square

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square for a quadratic rearranges $ax^2 + bx + c$ into the form $p(x + q)^2 + r$
- If $a \neq 1$, then factorise using a as a common factor.

Examples

Example 1 Complete the square for the quadratic expression $x^2 + 6x - 2$

$x^2 + 6x - 2$ $= (x + 3)^2 - 9 - 2$ $= (x + 3)^2 - 11$	**1** Write $x^2 + bx + c$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$ **2** Simplify

Example 2 Write $2x^2 - 5x + 1$ in the form $p(x + q)^2 + r$

$2x^2 - 5x + 1$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$
$= 2\left(x^2 - \frac{5}{2}x\right) + 1$	**2** Now complete the square by writing $x^2 - \frac{5}{2}x$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$
$= 2\left[\left(x - \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 1$	
$= 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8} + 1$	**3** Expand the square brackets – don't forget to multiply $\left(\frac{5}{4}\right)^2$ by the factor of 2
$= 2\left(x - \frac{5}{4}\right)^2 - \frac{17}{8}$	**4** Simplify

Practice

1 Write the following quadratic expressions in the form $(x + p)^2 + q$

a $x^2 + 4x + 3$ **b** $x^2 - 10x - 3$

c $x^2 - 8x$ **d** $x^2 + 6x$

e $x^2 - 2x + 7$ **f** $x^2 + 3x - 2$

2 Write the following quadratic expressions in the form $p(x + q)^2 + r$

a $2x^2 - 8x - 16$ **b** $4x^2 - 8x - 16$

c $3x^2 + 12x - 9$ **d** $2x^2 + 6x - 8$

3 Complete the square.

a $2x^2 + 3x + 6$ **b** $3x^2 - 2x$

c $5x^2 + 3x$ **d** $3x^2 + 5x + 3$

Extend

4 Write $(25x^2 + 30x + 12)$ in the form $(ax + b)^2 + c$.

Answers

1 **a** $(x + 2)^2 - 1$ **b** $(x - 5)^2 - 28$

c $(x - 4)^2 - 16$ **d** $(x + 3)^2 - 9$

e $(x - 1)^2 + 6$ **f** $\left(x+\frac{3}{2}\right)^2-\frac{17}{4}$

2 **a** $2(x - 2)^2 - 24$ **b** $4(x - 1)^2 - 20$

c $3(x + 2)^2 - 21$ **d** $2\left(x+\frac{3}{2}\right)^2-\frac{25}{2}$

3 **a** $2\left(x+\frac{3}{4}\right)^2+\frac{39}{8}$ **b** $3\left(x-\frac{1}{3}\right)^2-\frac{1}{3}$

c $5\left(x+\frac{3}{10}\right)^2-\frac{9}{20}$ **d** $3\left(x+\frac{5}{6}\right)^2+\frac{11}{12}$

4 $(5x + 3)^2 + 3$

Solving quadratic equations by factorisation

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose products is ac.
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Examples

Example 1 Solve $5x^2 = 15x$

$5x^2 = 15x$ $5x^2 - 15x = 0$	**1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by x as this would lose the solution $x = 0$.
$5x(x - 3) = 0$	**2** Factorise the quadratic equation. $5x$ is a common factor.
So $5x = 0$ or $(x - 3) = 0$	**3** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = 0$ or $x = 3$	**4** Solve these two equations.

Example 2 Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$ $b = 7$, $ac = 12$	**1** Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3)
$x^2 + 4x + 3x + 12 = 0$	**2** Rewrite the b term ($7x$) using these two factors.
$x(x + 4) + 3(x + 4) = 0$	**3** Factorise the first two terms and the last two terms.
$(x + 4)(x + 3) = 0$	**4** $(x + 4)$ is a factor of both terms.
So $(x + 4) = 0$ or $(x + 3) = 0$	**5** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = -4$ or $x = -3$	**6** Solve these two equations.

Example 3 Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$ So $(3x + 4) = 0$ or $(3x - 4) = 0$ $x = -\frac{4}{3}$ or $x = \frac{4}{3}$	**1** Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$. **2** When two values multiply to make zero, at least one of the values must be zero. **3** Solve these two equations.

Example 4 Solve $2x^2 - 5x - 12 = 0$

$b = -5$, $ac = -24$ So $2x^2 - 8x + 3x - 12 = 0$ $2x(x - 4) + 3(x - 4) = 0$ $(x - 4)(2x + 3) = 0$ So $(x - 4) = 0$ or $(2x + 3) = 0$ $x = 4$ or $x = -\frac{3}{2}$	**1** Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. (-8 and 3) **2** Rewrite the b term ($-5x$) using these two factors. **3** Factorise the first two terms and the last two terms. **4** $(x - 4)$ is a factor of both terms. **5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations.

Practice

1 Solve

a $6x^2 + 4x = 0$ **b** $28x^2 - 21x = 0$

c $x^2 + 7x + 10 = 0$ **d** $x^2 - 5x + 6 = 0$

e $x^2 - 3x - 4 = 0$ **f** $x^2 + 3x - 10 = 0$

g $x^2 - 10x + 24 = 0$ **h** $x^2 - 36 = 0$

i $x^2 + 3x - 28 = 0$ **j** $x^2 - 6x + 9 = 0$

k $2x^2 - 7x - 4 = 0$ **l** $3x^2 - 13x - 10 = 0$

2 Solve

a $x^2 - 3x = 10$ **b** $x^2 - 3 = 2x$

c $x^2 + 5x = 24$ **d** $x^2 - 42 = x$

e $x(x + 2) = 2x + 25$ **f** $x^2 - 30 = 3x - 2$

g $x(3x + 1) = x^2 + 15$ **h** $3x(x - 1) = 2(x + 1)$

Hint

Get all terms onto one side of the equation.

Solving quadratic equations by completing the square

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square lets you write a quadratic equation in the form $p(x + q)^2 + r = 0$.

Examples

Example 5 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$x^2 + 6x + 4 = 0$	**1** Write $x^2 + bx + c = 0$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = 0$
$(x + 3)^2 - 9 + 4 = 0$	
$(x + 3)^2 - 5 = 0$	**2** Simplify.
$(x + 3)^2 = 5$	**3** Rearrange the equation to work out x. First, add 5 to both sides.
$x + 3 = \pm\sqrt{5}$	**4** Square root both sides. Remember that the square root of a value gives two answers.
$x = \pm\sqrt{5} - 3$	**5** Subtract 3 from both sides to solve the equation.
So $x = -\sqrt{5} - 3$ or $x = \sqrt{5} - 3$	**6** Write down both solutions.

Example 6 Solve $2x^2 - 7x + 4 = 0$. Give your solutions in surd form.

$2x^2 - 7x + 4 = 0$	
$2\left(x^2 - \frac{7}{2}x\right) + 4 = 0$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$
$2\left[\left(x - \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right] + 4 = 0$	**2** Now complete the square by writing $x^2 - \frac{7}{2}x$ in the form $\left(x+\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$
$2\left(x - \frac{7}{4}\right)^2 - \frac{49}{8} + 4 = 0$	**3** Expand the square brackets.
$2\left(x - \frac{7}{4}\right)^2 - \frac{17}{8} = 0$	**4** Simplify.

(continued on next page)

$2\left(x-\frac{7}{4}\right)^2=\frac{17}{8}$	**5** Rearrange the equation to work out x. First, add $\frac{17}{8}$ to both sides.
$\left(x-\frac{7}{4}\right)^2=\frac{17}{16}$	**6** Divide both sides by 2.
$x-\frac{7}{4}=\pm\frac{\sqrt{17}}{4}$	**7** Square root both sides. Remember that the square root of a value gives two answers.
$x=\pm\frac{\sqrt{17}}{4}+\frac{7}{4}$	**8** Add $\frac{7}{4}$ to both sides.
So $x=\frac{7}{4}-\frac{\sqrt{17}}{4}$ or $x=\frac{7}{4}+\frac{\sqrt{17}}{4}$	**9** Write down both the solutions.

Practice

3 Solve by completing the square.

a $x^2 - 4x - 3 = 0$ **b** $x^2 - 10x + 4 = 0$

c $x^2 + 8x - 5 = 0$ **d** $x^2 - 2x - 6 = 0$

e $2x^2 + 8x - 5 = 0$ **f** $5x^2 + 3x - 4 = 0$

4 Solve by completing the square.

a $(x - 4)(x + 2) = 5$

b $2x^2 + 6x - 7 = 0$

c $x^2 - 5x + 3 = 0$

Hint

Get all terms onto one side of the equation.

Solving quadratic equations by using the formula

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- If $b^2 - 4ac$ is negative then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for a, b and c.

Examples

Example 7 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$a = 1$, $b = 6$, $c = 4$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$	**2** Substitute $a = 1$, $b = 6$, $c = 4$ into the formula.
$x = \dfrac{-6 \pm \sqrt{20}}{2}$	**3** Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2.
$x = \dfrac{-6 \pm 2\sqrt{5}}{2}$	**4** Simplify $\sqrt{20}$. $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$
$x = -3 \pm \sqrt{5}$	**5** Simplify by dividing numerator and denominator by 2.
So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$	**6** Write down both the solutions.

Example 8 Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3, b = -7, c = -2$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c, making sure you get the signs right and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$	**2** Substitute $a = 3$, $b = -7$, $c = -2$ into the formula.
$x = \dfrac{7 \pm \sqrt{73}}{6}$	**3** Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2.
So $x = \dfrac{7 - \sqrt{73}}{6}$ or $x = \dfrac{7 + \sqrt{73}}{6}$	**4** Write down both the solutions.

Practice

5 Solve, giving your solutions in surd form.

a $3x^2 + 6x + 2 = 0$ **b** $2x^2 - 4x - 7 = 0$

6 Solve the equation $x^2 - 7x + 2 = 0$

Give your solutions in the form $\dfrac{a \pm \sqrt{b}}{c}$, where a, b and c are integers.

7 Solve $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

Hint

Get all terms onto one side of the equation.

Extend

8 Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

a $4x(x - 1) = 3x - 2$

b $10 = (x + 1)^2$

c $x(3x - 1) = 10$

Answers

1 **a** $x = 0$ or $x = -\frac{2}{3}$ **b** $x = 0$ or $x = \frac{3}{4}$

c $x = -5$ or $x = -2$ **d** $x = 2$ or $x = 3$

e $x = -1$ or $x = 4$ **f** $x = -5$ or $x = 2$

g $x = 4$ or $x = 6$ **h** $x = -6$ or $x = 6$

i $x = -7$ or $x = 4$ **j** $x = 3$

k $x = -\frac{1}{2}$ or $x = 4$ **l** $x = -\frac{2}{3}$ or $x = 5$

2 **a** $x = -2$ or $x = 5$ **b** $x = -1$ or $x = 3$

c $x = -8$ or $x = 3$ **d** $x = -6$ or $x = 7$

e $x = -5$ or $x = 5$ **f** $x = -4$ or $x = 7$

g $x = -3$ or $x = 2\frac{1}{2}$ **h** $x = -\frac{1}{3}$ or $x = 2$

3 **a** $x = 2 + \sqrt{7}$ or $x = 2 - \sqrt{7}$ **b** $x = 5 + \sqrt{21}$ or $x = 5 - \sqrt{21}$

c $x = -4 + \sqrt{21}$ or $x = -4 - \sqrt{21}$ **d** $x = 1 + \sqrt{7}$ or $x = 1 - \sqrt{7}$

e $x = -2 + \sqrt{6.5}$ or $x = -2 - \sqrt{6.5}$ **f** $x = \frac{-3+\sqrt{89}}{10}$ or $x = \frac{-3-\sqrt{89}}{10}$

4 **a** $x = 1 + \sqrt{14}$ or $x = 1 - \sqrt{14}$ **b** $x = \frac{-3+\sqrt{23}}{2}$ or $x = \frac{-3-\sqrt{23}}{2}$

c $x = \frac{5+\sqrt{13}}{2}$ or $x = \frac{5-\sqrt{13}}{2}$

5 **a** $x = -1 + \frac{\sqrt{3}}{3}$ or $x = -1 - \frac{\sqrt{3}}{3}$ **b** $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$

6 $x = \frac{7+\sqrt{41}}{2}$ or $x = \frac{7-\sqrt{41}}{2}$

7 $x = \frac{-3+\sqrt{89}}{20}$ or $x = \frac{-3-\sqrt{89}}{20}$

8 **a** $x = \frac{7+\sqrt{17}}{8}$ or $x = \frac{7-\sqrt{17}}{8}$

b $x = -1 + \sqrt{10}$ or $x = -1 - \sqrt{10}$

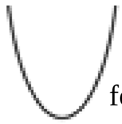
c $x = -1\frac{2}{3}$ or $x = 2$

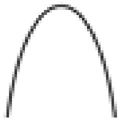
Sketching quadratic graphs

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- The graph of the quadratic function $y = ax^2 + bx + c$, where $a \neq 0$, is a curve called a parabola.
- Parabolas have a line of symmetry and a shape as shown.

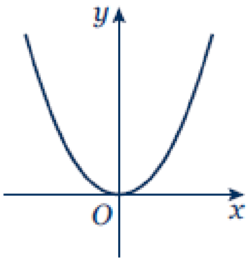
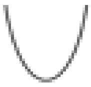

for $a > 0$


for $a < 0$

- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y-axis substitute $x = 0$ into the function.
- To find where the curve intersects the x-axis substitute $y = 0$ into the function.
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- To find the coordinates of the maximum or minimum point (turning points) of a quadratic curve (parabola) you can use the completed square form of the function.

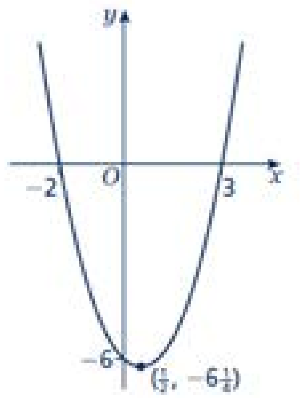
Examples

Example 1 Sketch the graph of $y = x^2$.

	The graph of $y = x^2$ is a parabola. When $x = 0$, $y = 0$. $a = 1$ which is greater than zero, so the graph has the shape: 

Example 2 Sketch the graph of $y = x^2 - x - 6$.

When $x = 0$, $y = 0^2 - 0 - 6 = -6$ So the graph intersects the y-axis at $(0, -6)$	**1** Find where the graph intersects the y-axis by substituting $x = 0$.
When $y = 0$, $x^2 - x - 6 = 0$	**2** Find where the graph intersects the x-axis by substituting $y = 0$.
$(x + 2)(x - 3) = 0$	**3** Solve the equation by factorising.
$x = -2$ or $x = 3$	**4** Solve $(x + 2) = 0$ and $(x - 3) = 0$.
So, the graph intersects the x-axis at $(-2, 0)$ and $(3, 0)$	**5** $a = 1$ which is greater than zero, so the graph has the shape: 
	(continued on next page)

$x^2 - x - 6 = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 6$ $= \left(x - \frac{1}{2}\right)^2 - \frac{25}{4}$	**6** To find the turning point, complete the square.
When $\left(x - \frac{1}{2}\right)^2 = 0$, $x = \frac{1}{2}$ and $y = -\frac{25}{4}$, so the turning point is at the point $\left(\frac{1}{2}, -\frac{25}{4}\right)$ 	**7** The turning point is the minimum value for this expression and occurs when the term in the bracket is equal to zero.

Practice

1 Sketch the graph of $y = -x^2$.

2 Sketch each graph, labelling where the curve crosses the axes.

a $y = (x + 2)(x - 1)$ **b** $y = x(x - 3)$ **c** $y = (x + 1)(x + 5)$

3 Sketch each graph, labelling where the curve crosses the axes.

a $y = x^2 - x - 6$ **b** $y = x^2 - 5x + 4$ **c** $y = x^2 - 4$

d $y = x^2 + 4x$ **e** $y = 9 - x^2$ **f** $y = x^2 + 2x - 3$

4 Sketch the graph of $y = 2x^2 + 5x - 3$, labelling where the curve crosses the axes.

Extend

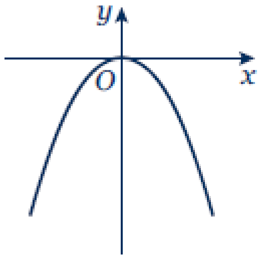
5 Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.

a $y = x^2 - 5x + 6$ **b** $y = -x^2 + 7x - 12$ **c** $y = -x^2 + 4x$

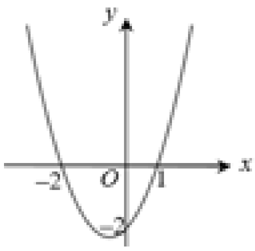
6 Sketch the graph of $y = x^2 + 2x + 1$. Label where the curve crosses the axes and write down the equation of the line of symmetry.

Answers

1



2 **a**



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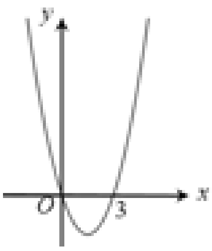

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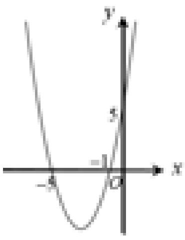
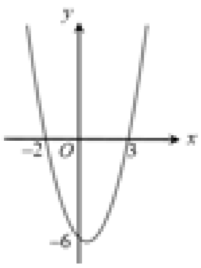


3 **a**



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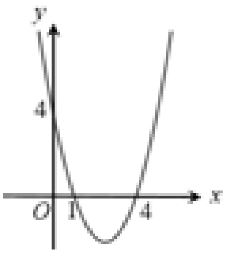

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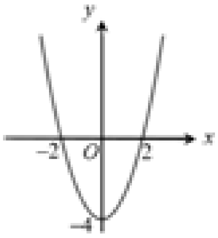

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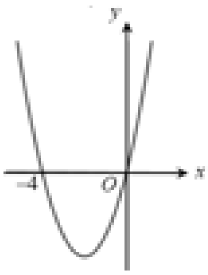

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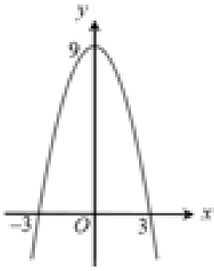

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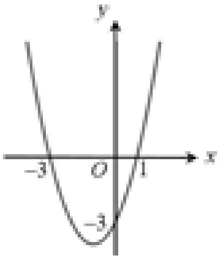

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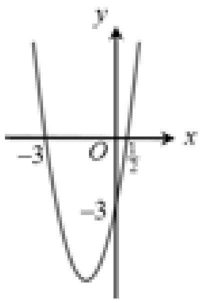
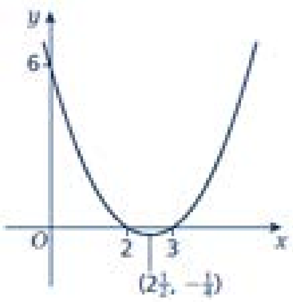


5 **a**



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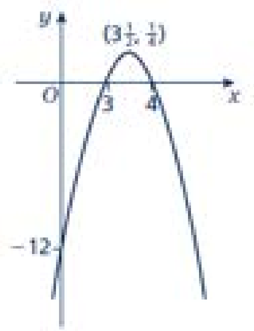

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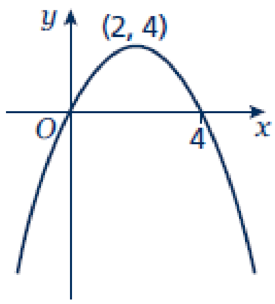

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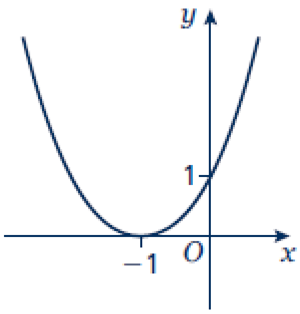


Line of symmetry at $x = -1$.

Solving linear simultaneous equations using the elimination method

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

Examples

Example 1 Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

$\begin{array}{rl} & 3x + y = 5 \\ - & x + y = 1 \\ \hline & 2x \quad\;\; = 4 \end{array}$ So $x = 2$	**1** Subtract the second equation from the first equation to eliminate the y term.
Using $x + y = 1$ $2 + y = 1$ So $y = -1$	**2** To find the value of y, substitute $x = 2$ into one of the original equations.
Check: equation 1: $3 \times 2 + (-1) = 5$ YES equation 2: $2 + (-1) = 1$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 2 Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

$\begin{array}{rl} & x + 2y = 13 \\ + & 5x - 2y = 5 \\ \hline & 6x \qquad = 18 \end{array}$ So $x = 3$	**1** Add the two equations together to eliminate the y term.
Using $x + 2y = 13$ $3 + 2y = 13$ So $y = 5$	**2** To find the value of y, substitute $x = 3$ into one of the original equations.
Check: equation 1: $3 + 2 \times 5 = 13$ YES equation 2: $5 \times 3 - 2 \times 5 = 5$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 3 Solve $2x + 3y = 2$ and $5x + 4y = 12$ simultaneously.

$(2x + 3y = 2) \times 4 \rightarrow \quad 8x + 12y = 8$ $(5x + 4y = 12) \times 3 \rightarrow \quad 15x + 12y = 36$ $7x \qquad = 28$ So $x = 4$	**1** Multiply the first equation by 4 and the second equation by 3 to make the coefficient of y the same for both equations. Then subtract the first equation from the second equation to eliminate the y term.
Using $2x + 3y = 2$ $2 \times 4 + 3y = 2$ So $y = -2$	**2** To find the value of y, substitute $x = 4$ into one of the original equations.
Check: equation 1: $2 \times 4 + 3 \times (-2) = 2$ YES equation 2: $5 \times 4 + 4 \times (-2) = 12$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

1 $4x + y = 8$
$x + y = 5$

2 $3x + y = 7$
$3x + 2y = 5$

3 $4x + y = 3$
$3x - y = 11$

4 $3x + 4y = 7$
$x - 4y = 5$

5 $2x + y = 11$
$x - 3y = 9$

6 $2x + 3y = 11$
$3x + 2y = 4$

Solving linear simultaneous equations using the substitution method

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous
Textbook: Pure Year 1, 3.1 Linear simultaneous equations

Key points

- The subsitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

Examples

Example 4 Solve the simultaneous equations $y = 2x + 1$ and $5x + 3y = 14$

$5x + 3(2x + 1) = 14$	**1** Substitute $2x + 1$ for y into the second equation.
$5x + 6x + 3 = 14$ $11x + 3 = 14$	**2** Expand the brackets and simplify.
$11x = 11$ So $x = 1$	**3** Work out the value of x.
Using $y = 2x + 1$ $y = 2 \times 1 + 1$ So $y = 3$	**4** To find the value of y, substitute $x = 1$ into one of the original equations.
Check: equation 1: $3 = 2 \times 1 + 1$ YES equation 2: $5 \times 1 + 3 \times 3 = 14$ YES	**5** Substitute the values of x and y into both equations to check your answers.

Example 5 Solve $2x - y = 16$ and $4x + 3y = -3$ simultaneously.

$y = 2x - 16$	**1** Rearrange the first equation.
$4x + 3(2x - 16) = -3$	**2** Substitute $2x - 16$ for y into the second equation.
$4x + 6x - 48 = -3$ $10x - 48 = -3$	**3** Expand the brackets and simplify.
$10x = 45$ So $x = 4\frac{1}{2}$	**4** Work out the value of x.
Using $y = 2x - 16$ $y = 2 \times 4\frac{1}{2} - 16$ So $y = -7$	**5** To find the value of y, substitute $x = 4\frac{1}{2}$ into one of the original equations.
Check: equation 1: $2 \times 4\frac{1}{2} - (-7) = 16$ YES equation 2: $4 \times 4\frac{1}{2} + 3 \times (-7) = -3$ YES	**6** Substitute the values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

7 $y = x - 4$
$2x + 5y = 43$

8 $y = 2x - 3$
$5x - 3y = 11$

9 $2y = 4x + 5$
$9x + 5y = 22$

10 $2x = y - 2$
$8x - 5y = -11$

11 $3x + 4y = 8$
$2x - y = -13$

12 $3y = 4x - 7$
$2y = 3x - 4$

13 $3x = y - 1$
$2y - 2x = 3$

14 $3x + 2y + 1 = 0$
$4y = 8 - x$

Extend

15 Solve the simultaneous equations $3x + 5y - 20 = 0$ and $2(x + y) = \dfrac{3(y - x)}{4}$.

Answers

1 $x = 1, y = 4$

2 $x = 3, y = -2$

3 $x = 2, y = -5$

4 $x = 3, y = -\frac{1}{2}$

5 $x = 6, y = -1$

6 $x = -2, y = 5$

7 $x = 9, y = 5$

8 $x = -2, y = -7$

9 $x = \frac{1}{2}, y = 3\frac{1}{2}$

10 $x = \frac{1}{2}, y = 3$

11 $x = -4, y = 5$

12 $x = -2, y = -5$

13 $x = \frac{1}{4}, y = 1\frac{3}{4}$

14 $x = -2, y = 2\frac{1}{2}$

15 $x = -2\frac{1}{2}, y = 5\frac{1}{2}$

Solving linear and quadratic simultaneous equations

A LEVEL LINKS **Scheme of work:** 1c. Equations – quadratic/linear simultaneous

Key points

- Make one of the unknowns the subject of the linear equation (rearranging where necessary).
- Use the linear equation to substitute into the quadratic equation.
- There are usually two pairs of solutions.

Examples

Example 1 Solve the simultaneous equations $y = x + 1$ and $x^2 + y^2 = 13$

$x^2 + (x + 1)^2 = 13$	**1** Substitute $x + 1$ for y into the second equation.
$x^2 + x^2 + x + x + 1 = 13$ $2x^2 + 2x + 1 = 13$	**2** Expand the brackets and simplify.
$2x^2 + 2x - 12 = 0$ $(2x - 4)(x + 3) = 0$	**3** Factorise the quadratic equation.
So $x = 2$ or $x = -3$	**4** Work out the values of x.
Using $y = x + 1$ When $x = 2$, $y = 2 + 1 = 3$ When $x = -3$, $y = -3 + 1 = -2$	**5** To find the value of y, substitute both values of x into one of the original equations.
So the solutions are $x = 2,\ y = 3$ and $x = -3,\ y = -2$	
Check: equation 1: $3 = 2 + 1$ YES and $-2 = -3 + 1$ YES equation 2: $2^2 + 3^2 = 13$ YES and $(-3)^2 + (-2)^2 = 13$ YES	**6** Substitute both pairs of values of x and y into both equations to check your answers.

Example 2 Solve $2x + 3y = 5$ and $2y^2 + xy = 12$ simultaneously.

$x = \frac{5-3y}{2}$	**1** Rearrange the first equation.
$2y^2 + \left(\frac{5-3y}{2}\right)y = 12$	**2** Substitute $\frac{5-3y}{2}$ for x into the second equation. Notice how it is easier to substitute for x than for y.
$2y^2 + \frac{5y-3y^2}{2} = 12$ $4y^2 + 5y - 3y^2 = 24$	**3** Expand the brackets and simplify.
$y^2 + 5y - 24 = 0$ $(y + 8)(y - 3) = 0$	**4** Factorise the quadratic equation.
So $y = -8$ or $y = 3$	**5** Work out the values of y.
Using $2x + 3y = 5$ When $y = -8$, $2x + 3 \times (-8) = 5$, $x = 14.5$ When $y = 3$, $2x + 3 \times 3 = 5$, $x = -2$ So the solutions are $x = 14.5$, $y = -8$ and $x = -2$, $y = 3$	**6** To find the value of x, substitute both values of y into one of the original equations.
Check: equation 1: $2 \times 14.5 + 3 \times (-8) = 5$ YES and $2 \times (-2) + 3 \times 3 = 5$ YES equation 2: $2\times(-8)^2 + 14.5\times(-8) = 12$ YES and $2 \times (3)^2 + (-2) \times 3 = 12$ YES	**7** Substitute both pairs of values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

1 $y = 2x + 1$
$x^2 + y^2 = 10$

2 $y = 6 - x$
$x^2 + y^2 = 20$

3 $y = x - 3$
$x^2 + y^2 = 5$

4 $y = 9 - 2x$
$x^2 + y^2 = 17$

5 $y = 3x - 5$
$y = x^2 - 2x + 1$

6 $y = x - 5$
$y = x^2 - 5x - 12$

7 $y = x + 5$
$x^2 + y^2 = 25$

8 $y = 2x - 1$
$x^2 + xy = 24$

9 $y = 2x$
$y^2 - xy = 8$

10 $2x + y = 11$
$xy = 15$

Extend

11 $x - y = 1$
$x^2 + y^2 = 3$

12 $y - x = 2$
$x^2 + xy = 3$

Answers

1 $x = 1, y = 3$

$x = -\frac{9}{5},\ y = -\frac{13}{5}$

2 $x = 2, y = 4$

$x = 4, y = 2$

3 $x = 1, y = -2$

$x = 2, y = -1$

4 $x = 4, y = 1$

$x = \frac{16}{5},\ y = \frac{13}{5}$

5 $x = 3, y = 4$

$x = 2, y = 1$

6 $x = 7, y = 2$

$x = -1, y = -6$

7 $x = 0, y = 5$

$x = -5, y = 0$

8 $x = -\frac{8}{3}, y = -\frac{19}{3}$

$x = 3, y = 5$

9 $x = -2, y = -4$

$x = 2, y = 4$

10 $x = \frac{5}{2}, y = 6$

$x = 3, y = 5$

11 $x = \frac{1+\sqrt{5}}{2}, y = \frac{-1+\sqrt{5}}{2}$

$x = \frac{1-\sqrt{5}}{2}, y = \frac{-1-\sqrt{5}}{2}$

12 $x = \frac{-1+\sqrt{7}}{2}, y = \frac{3+\sqrt{7}}{2}$

$x = \frac{-1-\sqrt{7}}{2}, y = \frac{3-\sqrt{7}}{2}$

Solving simultaneous equations graphically

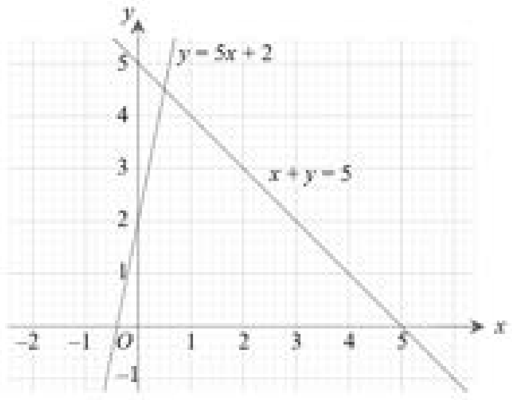
A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- You can solve any pair of simultaneous equations by drawing the graph of both equations and finding the point/points of intersection.

Examples

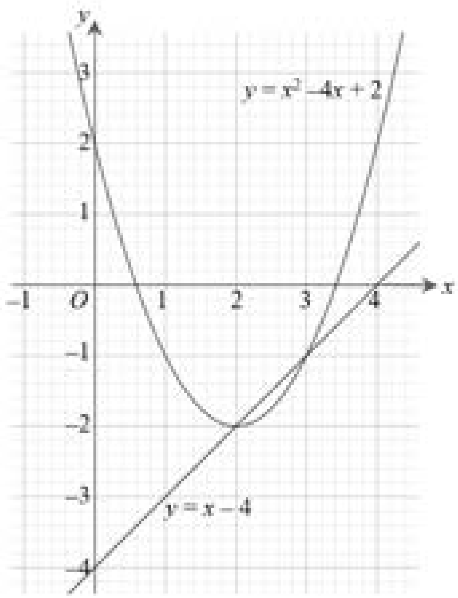
Example 1 Solve the simultaneous equations $y = 5x + 2$ and $x + y = 5$ graphically.

$y = 5 - x$	**1** Rearrange the equation $x + y = 5$ to make y the subject.
$y = 5 - x$ has gradient –1 and y-intercept 5. $y = 5x + 2$ has gradient 5 and y-intercept 2.  	**2** Plot both graphs on the same grid using the gradients and y-intercepts.
Lines intersect at $x = 0.5$, $y = 4.5$	**3** The solutions of the simultaneous equations are the point of intersection.
Check: First equation $y = 5x + 2$: $4.5 = 5 \times 0.5 + 2$ YES Second equation $x + y = 5$: $0.5 + 4.5 = 5$ YES	**4** Check your solutions by substituting the values into both equations.

Example 2 Solve the simultaneous equations $y = x - 4$ and $y = x^2 - 4x + 2$ graphically.

x	0	1	2	3	4
y	2	–1	–2	–1	2

1 Construct a table of values and calculate the points for the quadratic equation.



2 Plot the graph.

3 Plot the linear graph on the same grid using the gradient and y-intercept.
$y = x – 4$ has gradient 1 and y-intercept –4.

The line and curve intersect at
$x = 3$, $y = -1$ and $x = 2$, $y = -2$

4 The solutions of the simultaneous equations are the points of intersection.

Check:
First equation $y = x - 4$:
$-1 = 3 - 4$ YES
$-2 = 2 - 4$ YES
Second equation $y = x^2 - 4x + 2$:
$-1 = 3^2 - 4 \times 3 + 2$ YES
$-2 = 2^2 - 4 \times 2 + 2$ YES

5 Check your solutions by substituting the values into both equations.

Practice

1 Solve these pairs of simultaneous equations graphically.

a $y = 3x - 1$ and $y = x + 3$

b $y = x - 5$ and $y = 7 - 5x$

c $y = 3x + 4$ and $y = 2 - x$

2 Solve these pairs of simultaneous equations graphically.

a $x + y = 0$ and $y = 2x + 6$

b $4x + 2y = 3$ and $y = 3x - 1$

c $2x + y + 4 = 0$ and $2y = 3x - 1$

Hint

Rearrange the equation to make y the subject.

3 Solve these pairs of simultaneous equations graphically.

a $y = x - 1$ and $y = x^2 - 4x + 3$

b $y = 1 - 3x$ and $y = x^2 - 3x - 3$

c $y = 3 - x$ and $y = x^2 + 2x + 5$

4 Solve the simultaneous equations $x + y = 1$ and $x^2 + y^2 = 25$ graphically.

Extend

5 **a** Solve the simultaneous equations $2x + y = 3$ and $x^2 + y = 4$

i graphically

ii algebraically to 2 decimal places.

b Which method gives the more accurate solutions? Explain your answer.

Answers

1 **a** $x = 2, y = 5$
b $x = 2, y = -3$
c $x = -0.5, y = 2.5$

2 **a** $x = -2, y = 2$
b $x = 0.5, y = 0.5$
c $x = -1, y = -2$

3 **a** $x = 1, y = 0$ and $x = 4, y = 3$
b $x = -2, y = 7$ and $x = 2, y = -5$
c $x = -2, y = 5$ and $x = -1, y = 4$

4 $x = -3, y = 4$ and $x = 4, y = -3$

5 **a** **i** $x = 2.5, y = -2$ and $x = -0.5, y = 4$
ii $x = 2.41, y = -1.83$ and $x = -0.41, y = 3.83$
b Solving algebraically gives the more accurate solutions as the solutions from the graph are only estimates, based on the accuracy of your graph.

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Linear inequalities

A LEVEL LINKS
Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. < becomes >.

Examples

Example 1 Solve $-8 \leq 4x < 16$

$-8 \leq 4x < 16$ $-2 \leq x < 4$	Divide all three terms by 4.

Example 2 Solve $4 \leq 5x < 10$

$4 \leq 5x < 10$ $\frac{4}{5} \leq x < 2$	Divide all three terms by 5.

Example 3 Solve $2x - 5 < 7$

$2x - 5 < 7$ $2x < 12$ $x < 6$	**1** Add 5 to both sides. **2** Divide both sides by 2.

Example 4 Solve $2 - 5x \geq -8$

$2 - 5x \geq -8$ $-5x \geq -10$ $x \leq 2$	**1** Subtract 2 from both sides. **2** Divide both sides by −5. Remember to reverse the inequality when dividing by a negative number.

Example 5 Solve $4(x - 2) > 3(9 - x)$

$4(x - 2) > 3(9 - x)$ $4x - 8 > 27 - 3x$ $7x - 8 > 27$ $7x > 35$ $x > 5$	**1** Expand the brackets. **2** Add $3x$ to both sides. **3** Add 8 to both sides. **4** Divide both sides by 7.

Practice

1 Solve these inequalities.

a $4x > 16$ **b** $5x - 7 \leq 3$ **c** $1 \geq 3x + 4$

d $5 - 2x < 12$ **e** $\frac{x}{2} \geq 5$ **f** $8 < 3 - \frac{x}{3}$

2 Solve these inequalities.

a $\frac{x}{5} < -4$ **b** $10 \geq 2x + 3$ **c** $7 - 3x > -5$

3 Solve

a $2 - 4x \geq 18$ **b** $3 \leq 7x + 10 < 45$ **c** $6 - 2x \geq 4$

d $4x + 17 < 2 - x$ **e** $4 - 5x < -3x$ **f** $-4x \geq 24$

4 Solve these inequalities.

a $3t + 1 < t + 6$ **b** $2(3n - 1) \geq n + 5$

5 Solve.

a $3(2 - x) > 2(4 - x) + 4$ **b** $5(4 - x) > 3(5 - x) + 2$

Extend

6 Find the set of values of x for which $2x + 1 > 11$ and $4x - 2 > 16 - 2x$.

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Answers

1 **a** $x > 4$ **b** $x \le 2$ **c** $x \le -1$

d $x > -\frac{7}{2}$ **e** $x \ge 10$ **f** $x < -15$

2 **a** $x < -20$ **b** $x \le 3.5$ **c** $x < 4$

3 **a** $x \le -4$ **b** $-1 \le x < 5$ **c** $x \le 1$

d $x < -3$ **e** $x > 2$ **f** $x \le -6$

4 **a** $t < \frac{5}{2}$ **b** $n \ge \frac{7}{5}$

5 **a** $x < -6$ **b** $x < \frac{3}{2}$

6 $x > 5$ (which also satisfies $x > 3$)

Quadratic inequalities

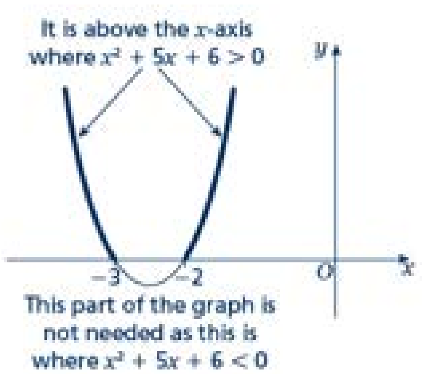
A LEVEL LINKS
Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

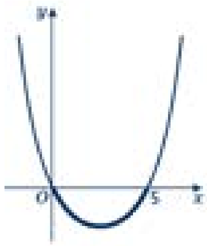
- First replace the inequality sign by = and solve the quadratic equation.
- Sketch the graph of the quadratic function.
- Use the graph to find the values which satisfy the quadratic inequality.

Examples

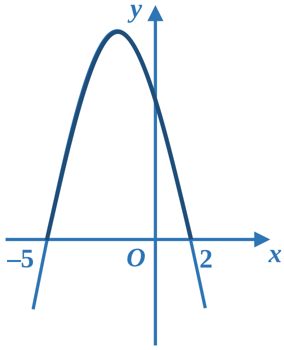
Example 1 Find the set of values of x which satisfy $x^2 + 5x + 6 > 0$

$x^2 + 5x + 6 = 0$ $(x + 3)(x + 2) = 0$ $x = -3$ or $x = -2$	**1** Solve the quadratic equation by factorising.
	**2** Sketch the graph of $y = (x + 3)(x + 2)$ **3** Identify on the graph where $x^2 + 5x + 6 > 0$, i.e. where $y > 0$
$x < -3$ or $x > -2$	**4** Write down the values which satisfy the inequality $x^2 + 5x + 6 > 0$

Example 2 Find the set of values of x which satisfy $x^2 - 5x \leq 0$

$x^2 - 5x = 0$ $x(x - 5) = 0$ $x = 0$ or $x = 5$	**1** Solve the quadratic equation by factorising.
	**2** Sketch the graph of $y = x(x - 5)$ **3** Identify on the graph where $x^2 - 5x \leq 0$, i.e. where $y \leq 0$
$0 \leq x \leq 5$	**4** Write down the values which satisfy the inequality $x^2 - 5x \leq 0$

Example 3 Find the set of values of x which satisfy $-x^2 - 3x + 10 \geq 0$

$-x^2 - 3x + 10 = 0$ $(-x + 2)(x + 5) = 0$ $x = 2$ or $x = -5$	**1** Solve the quadratic equation by factorising.
	**2** Sketch the graph of $y = (-x + 2)(x + 5) = 0$ **3** Identify on the graph where $-x^2 - 3x + 10 \geq 0$, i.e. where $y \geq 0$
$-5 \leq x \leq 2$	**3** Write down the values which satisfy the inequality $-x^2 - 3x + 10 \geq 0$

Practice

1 Find the set of values of x for which $(x + 7)(x - 4) \leq 0$

2 Find the set of values of x for which $x^2 - 4x - 12 \geq 0$

3 Find the set of values of x for which $2x^2 - 7x + 3 < 0$

4 Find the set of values of x for which $4x^2 + 4x - 3 > 0$

5 Find the set of values of x for which $12 + x - x^2 \geq 0$

Extend

Find the set of values which satisfy the following inequalities.

6 $x^2 + x \leq 6$

7 $x(2x - 9) < -10$

8 $6x^2 \geq 15 + x$

Answers

1 $-7 \le x \le 4$

2 $x \le -2$ or $x \ge 6$

3 $\frac{1}{2} < x < 3$

4 $x < -\frac{3}{2}$ or $x > \frac{1}{2}$

5 $-3 \le x \le 4$

6 $-3 \le x \le 2$

7 $2 < x < 2\frac{1}{2}$

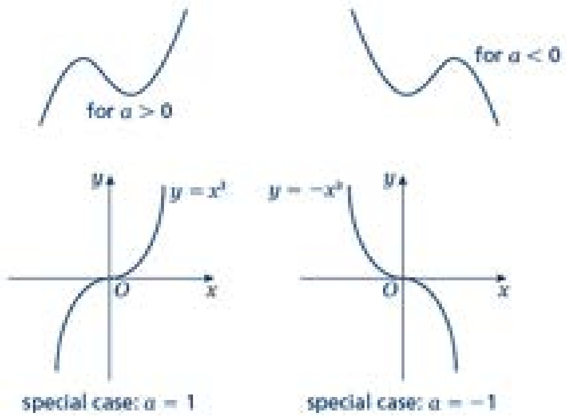
8 $x \le -\frac{3}{2}$ or $x \ge \frac{5}{3}$

Sketching cubic and reciprocal graphs

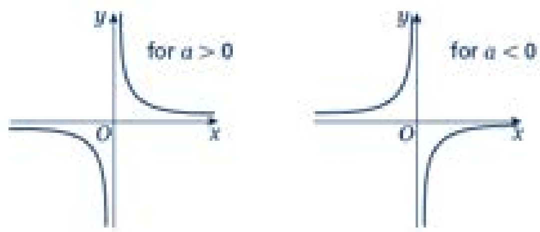
A LEVEL LINKS
Scheme of work: 1e. Graphs – cubic, quartic and reciprocal

Key points

- The graph of a cubic function, which can be written in the form $y = ax^3 + bx^2 + cx + d$, where $a \neq 0$, has one of the shapes shown here.



- The graph of a reciprocal function of the form $y = \frac{a}{x}$ has one of the shapes shown here.



- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y-axis substitute $x = 0$ into the function.
- To find where the curve intersects the x-axis substitute $y = 0$ into the function.
- Where appropriate, mark and label the asymptotes on the graph.
- Asymptotes are lines (usually horizontal or vertical) which the curve gets closer to but never touches or crosses. Asymptotes usually occur with reciprocal functions. For example, the asymptotes for the graph of $y = \frac{a}{x}$ are the two axes (the lines $y = 0$ and $x = 0$).
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- A double root is when two of the solutions are equal. For example $(x - 3)^2(x + 2)$ has a double root at $x = 3$.
- When there is a double root, this is one of the turning points of a cubic function.

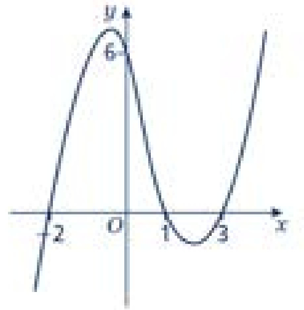
Examples

Example 1 Sketch the graph of $y = (x - 3)(x - 1)(x + 2)$

To sketch a cubic curve find intersects with both axes and use the key points above for the correct shape.

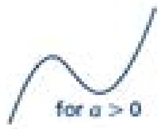
When $x = 0$, $y = (0 - 3)(0 - 1)(0 + 2)$
$= (-3) \times (-1) \times 2 = 6$
The graph intersects the y-axis at $(0, 6)$

When $y = 0$, $(x - 3)(x - 1)(x + 2) = 0$
So $x = 3$, $x = 1$ or $x = -2$
The graph intersects the x-axis at
$(-2, 0)$, $(1, 0)$ and $(3, 0)$



1 Find where the graph intersects the axes by substituting $x = 0$ and $y = 0$. Make sure you get the coordinates the right way around, (x, y).
2 Solve the equation by solving $x - 3 = 0$, $x - 1 = 0$ and $x + 2 = 0$

3 Sketch the graph.
$a = 1 > 0$ so the graph has the shape:



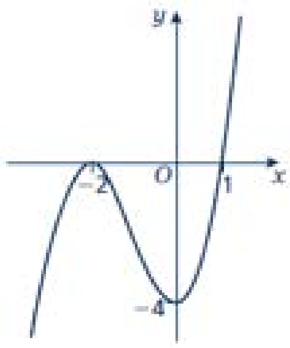
Example 2 Sketch the graph of $y = (x + 2)^2(x - 1)$

To sketch a cubic curve find intersects with both axes and use the key points above for the correct shape.

When $x = 0$, $y = (0 + 2)^2(0 - 1)$
$= 2^2 \times (-1) = -4$
The graph intersects the y-axis at $(0, -4)$

When $y = 0$, $(x + 2)^2(x - 1) = 0$
So $x = -2$ or $x = 1$

$(-2, 0)$ is a turning point as $x = -2$ is a double root.
The graph crosses the x-axis at $(1, 0)$



1 Find where the graph intersects the axes by substituting $x = 0$ and $y = 0$.

2 Solve the equation by solving $x + 2 = 0$ and $x - 1 = 0$

3 $a = 1 > 0$ so the graph has the shape:

Practice

1 Here are six equations.

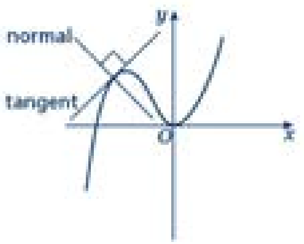
A $y = \dfrac{5}{x}$ **B** $y = x^2 + 3x - 10$ **C** $y = x^3 + 3x^2$

D $y = 1 - 3x^2 - x^3$ **E** $y = x^3 - 3x^2 - 1$ **F** $x + y = 5$

> **Hint**
>
> Find where each of the cubic equations cross the y-axis.

Here are six graphs.

i



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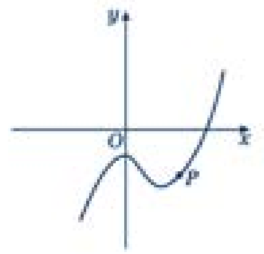

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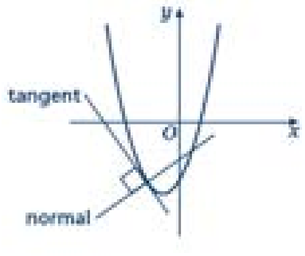

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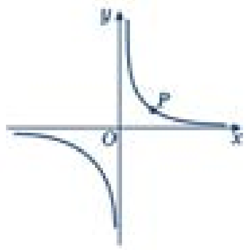

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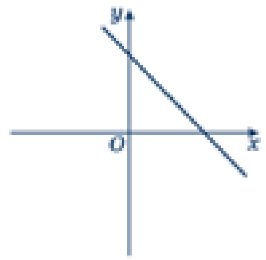
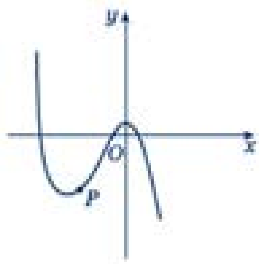


vi



a Match each graph to its equation.

b Copy the graphs ii, iv and vi and draw the tangent and normal each at point P.

Sketch the following graphs

2 $y = 2x^3$

3 $y = x(x - 2)(x + 2)$

4 $y = (x + 1)(x + 4)(x - 3)$

5 $y = (x + 1)(x - 2)(1 - x)$

6 $y = (x - 3)^2(x + 1)$

7 $y = (x - 1)^2(x - 2)$

8 $y = \dfrac{3}{x}$

> **Hint:** Look at the shape of $y = \dfrac{a}{x}$ in the second key point.

9 $y = -\dfrac{2}{x}$

Extend

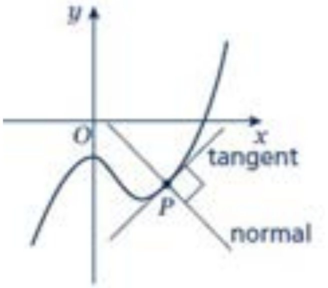
10 Sketch the graph of $y = \dfrac{1}{x+2}$

11 Sketch the graph of $y = \dfrac{1}{x-1}$

Answers

1 **a** i – C
ii – E
iii – B
iv – A
v – F
vi – D

b **ii**



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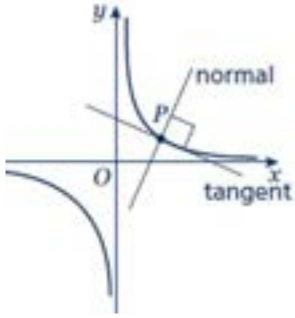

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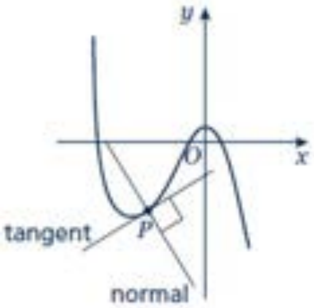

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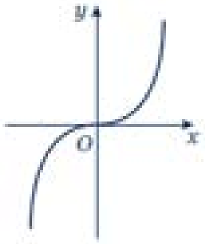

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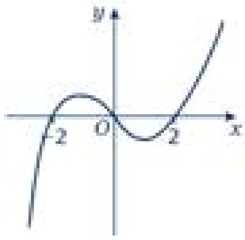

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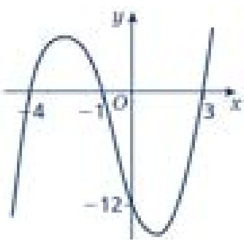

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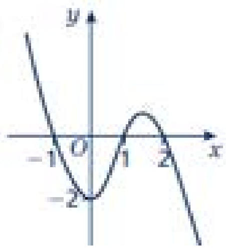

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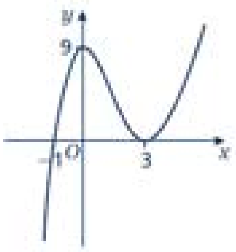

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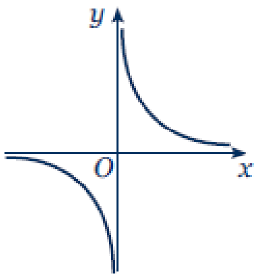

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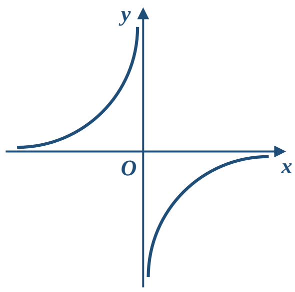

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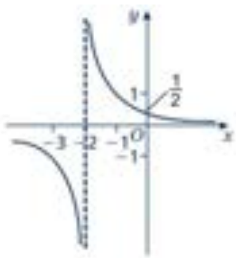
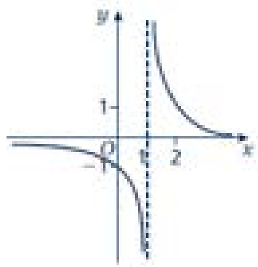


11



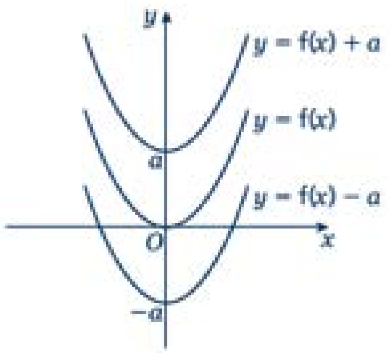
Translating graphs

A LEVEL LINKS
Scheme of work: 1f. Transformations – transforming graphs – f(x) notation

Key points

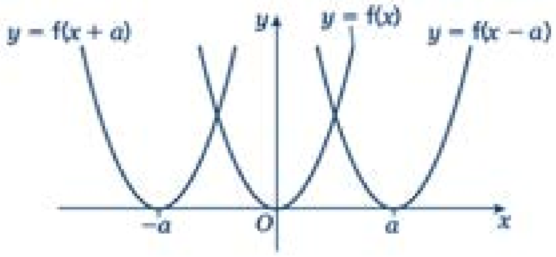
- The transformation $y = \text{f}(x) \pm a$ is a translation of $y = \text{f}(x)$ parallel to the y-axis; it is a vertical translation.

  As shown on the graph,
  - $y = \text{f}(x) + a$ translates $y = \text{f}(x)$ up
  - $y = \text{f}(x) – a$ translates $y = \text{f}(x)$ down.



- The transformation $y = \text{f}(x \pm a)$ is a translation of $y = \text{f}(x)$ parallel to the x-axis; it is a horizontal translation.

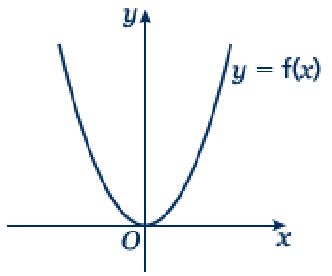
  As shown on the graph,
  - $y = \text{f}(x + a)$ translates $y = \text{f}(x)$ to the left
  - $y = \text{f}(x – a)$ translates $y = \text{f}(x)$ to the right.

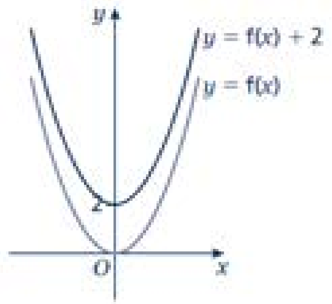


Examples

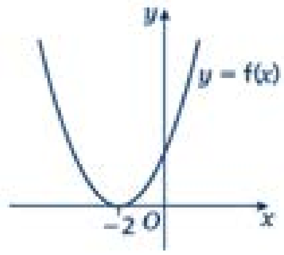
Example 1 The graph shows the function $y = \text{f}(x)$.

Sketch the graph of $y = \text{f}(x) + 2$.

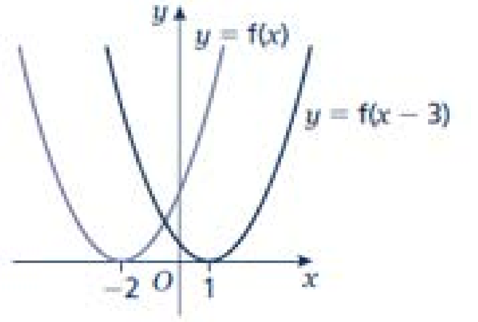


	For the function $y = \text{f}(x) + 2$ translate the function $y = \text{f}(x)$ 2 units up.

Example 2 The graph shows the function $y = \text{f}(x)$.

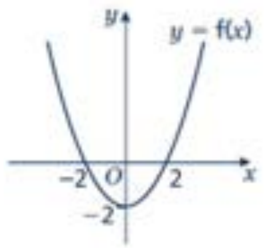


Sketch the graph of $y = \mathrm{f}(x - 3)$.

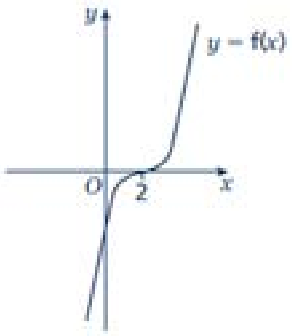
	For the function $y = \mathrm{f}(x - 3)$ translate the function $y = \mathrm{f}(x)$ 3 units right.

Practice

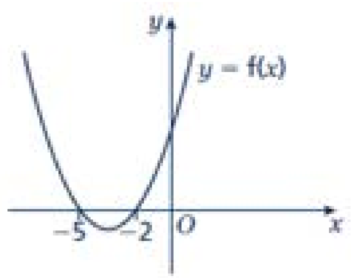
1 The graph shows the function $y = \mathrm{f}(x)$.
Copy the graph and on the same axes sketch and label the graphs of $y = \mathrm{f}(x) + 4$ and $y = \mathrm{f}(x + 2)$.



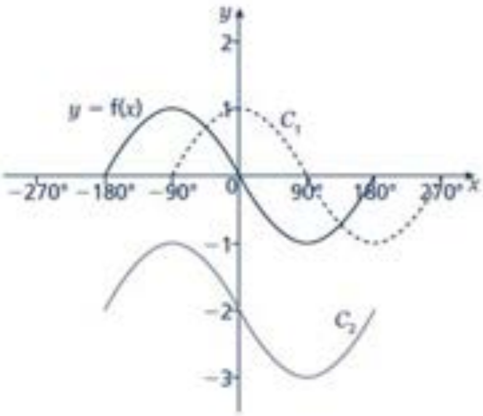
2 The graph shows the function $y = \mathrm{f}(x)$.
Copy the graph and on the same axes sketch and label the graphs of $y = \mathrm{f}(x + 3)$ and $y = \mathrm{f}(x) - 3$.



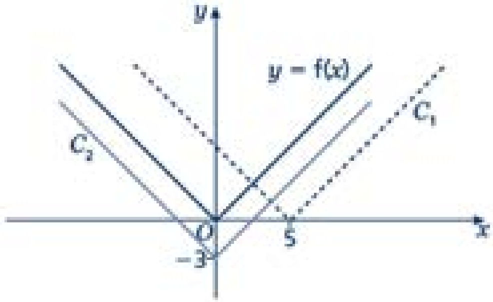
3 The graph shows the function $y = \mathrm{f}(x)$.
Copy the graph and on the same axes sketch the graph of $y = \mathrm{f}(x - 5)$.



4 The graph shows the function $y = \text{f}(x)$ and two transformations of $y = \text{f}(x)$, labelled C_1 and C_2. Write down the equations of the translated curves C_1 and C_2 in function form.



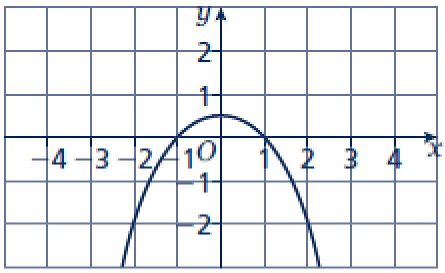
5 The graph shows the function $y = \text{f}(x)$ and two transformations of $y = \text{f}(x)$, labelled C_1 and C_2. Write down the equations of the translated curves C_1 and C_2 in function form.



6 The graph shows the function $y = \text{f}(x)$.

a Sketch the graph of $y = \text{f}(x) + 2$

b Sketch the graph of $y = \text{f}(x + 2)$

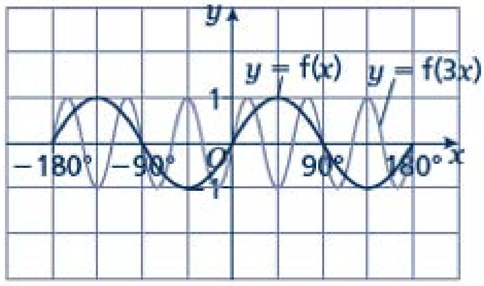


Stretching graphs

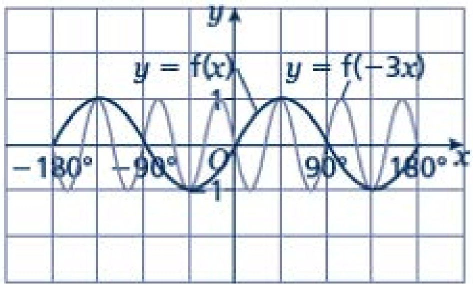
A LEVEL LINKS
Scheme of work: 1f. Transformations – transforming graphs – f(x) notation
Textbook: Pure Year 1, 4.6 Stretching graphs

Key points

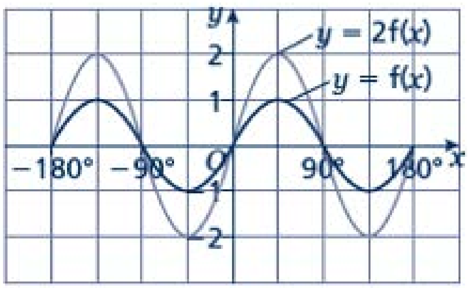
- The transformation $y = f(ax)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{a}$ parallel to the x-axis.



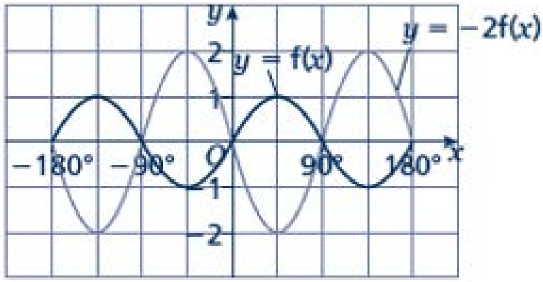
- The transformation $y = f(–ax)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{a}$ parallel to the x-axis and then a reflection in the y-axis.



- The transformation $y = af(x)$ is a vertical stretch of $y = f(x)$ with scale factor a parallel to the y-axis.



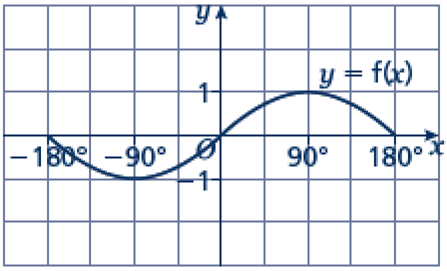
- The transformation $y = –af(x)$ is a vertical stretch of $y = f(x)$ with scale factor a parallel to the y-axis and then a reflection in the x-axis.

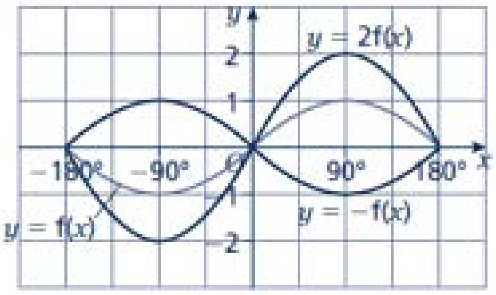


Examples

Example 3 The graph shows the function $y = \mathrm{f}(x)$.

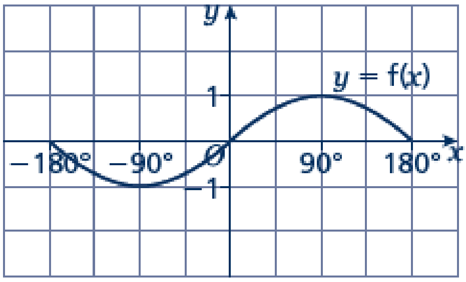
Sketch and label the graphs of
$y = 2\mathrm{f}(x)$ and $y = -\mathrm{f}(x)$.

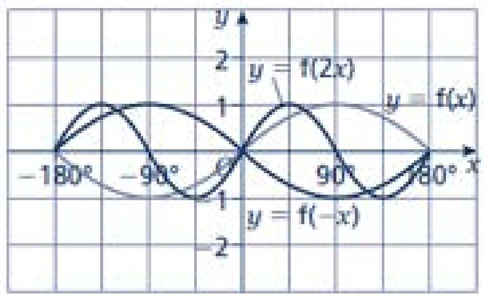


	The function $y = 2\mathrm{f}(x)$ is a vertical stretch of $y = \mathrm{f}(x)$ with scale factor 2 parallel to the y-axis. The function $y = -\mathrm{f}(x)$ is a reflection of $y = \mathrm{f}(x)$ in the x-axis.

Example 4 The graph shows the function $y = \mathrm{f}(x)$.

Sketch and label the graphs of
$y = \mathrm{f}(2x)$ and $y = \mathrm{f}(-x)$.



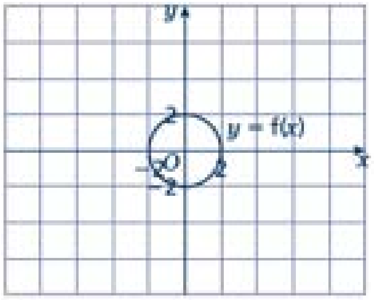
	The function $y = \mathrm{f}(2x)$ is a horizontal stretch of $y = \mathrm{f}(x)$ with scale factor $\frac{1}{2}$ parallel to the x-axis. The function $y = \mathrm{f}(-x)$ is a reflection of $y = \mathrm{f}(x)$ in the y-axis.

Practice

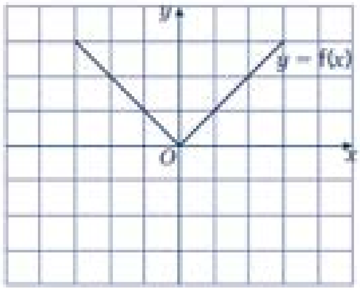
7 The graph shows the function $y = f(x)$.

a Copy the graph and on the same axes sketch and label the graph of $y = 3f(x)$.

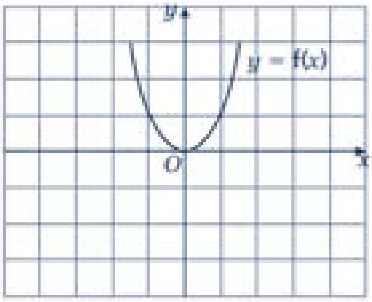
b Make another copy of the graph and on the same axes sketch and label the graph of $y = f(2x)$.



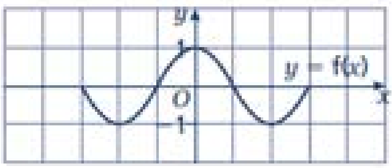
8 The graph shows the function $y = f(x)$. Copy the graph and on the same axes sketch and label the graphs of $y = –2f(x)$ and $y = f(3x)$.



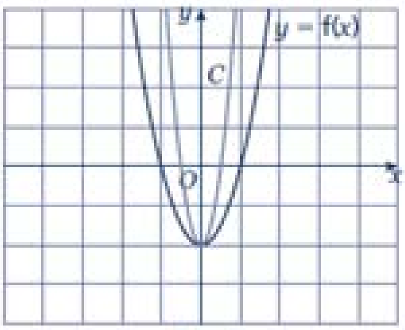
9 The graph shows the function $y = f(x)$. Copy the graph and, on the same axes, sketch and label the graphs of $y = –f(x)$ and $y = f\left(\frac{1}{2}x\right)$.



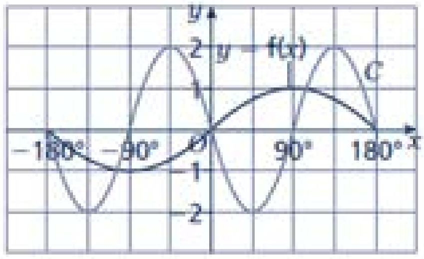
10 The graph shows the function $y = f(x)$. Copy the graph and, on the same axes, sketch the graph of $y = –f(2x)$.



11 The graph shows the function $y = f(x)$ and a transformation, labelled C. Write down the equation of the translated curve C in function form.



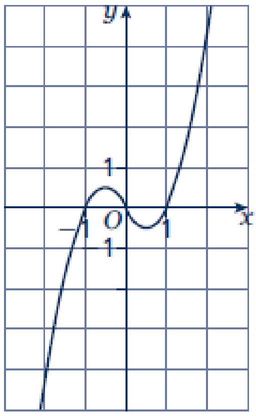
12 The graph shows the function $y = \mathrm{f}(x)$ and a transformation labelled C.
Write down the equation of the translated curve C in function form.



13 The graph shows the function $y = \mathrm{f}(x)$.

a Sketch the graph of $y = -\mathrm{f}(x)$.

b Sketch the graph of $y = 2\mathrm{f}(x)$.



Extend

14 **a** Sketch and label the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (x - 1)(x + 1)$.

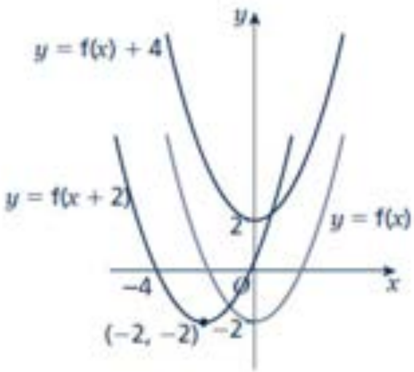
b On the same axes, sketch and label the graphs of $y = \mathrm{f}(x) - 2$ and $y = \mathrm{f}(x + 2)$.

15 **a** Sketch and label the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = -(x + 1)(x - 2)$.

b On the same axes, sketch and label the graph of $y = \mathrm{f}\left(-\frac{1}{2}x\right)$.

Answers

1



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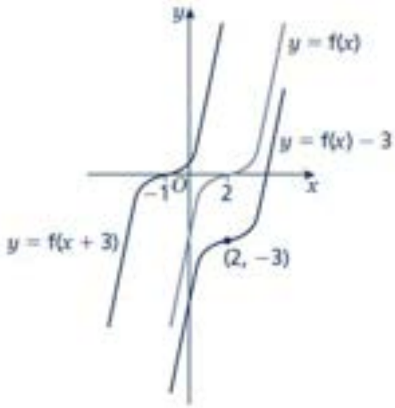

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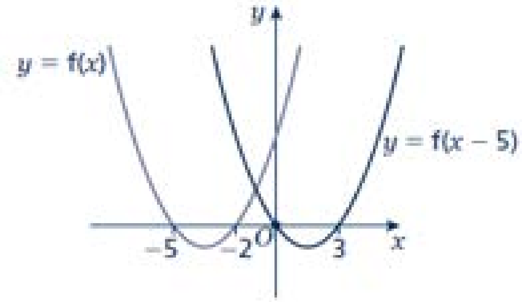
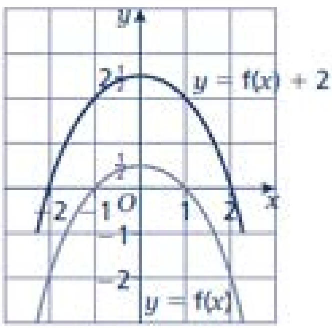


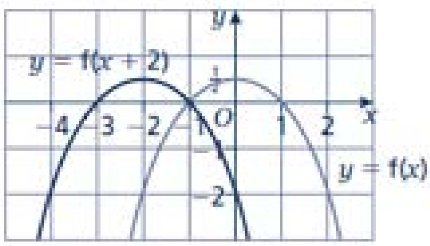
4 C_1: $y = \mathrm{f}(x - 90°)$
C_2: $y = \mathrm{f}(x) - 2$

5 C_1: $y = \mathrm{f}(x - 5)$
C_2: $y = \mathrm{f}(x) - 3$

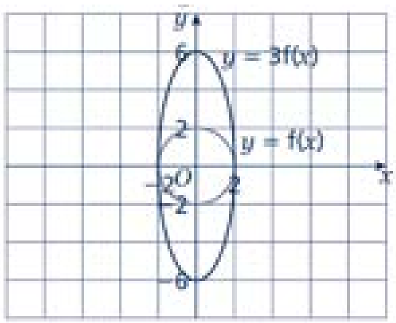
6 **a**



b



7 **a**



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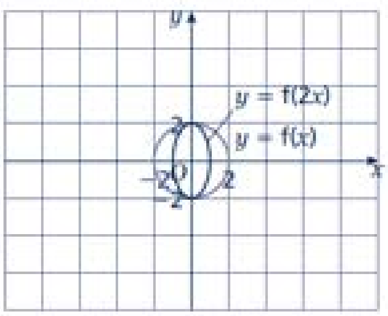

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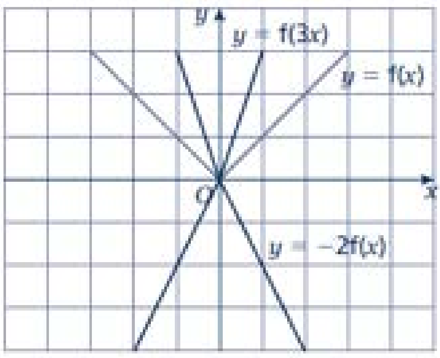

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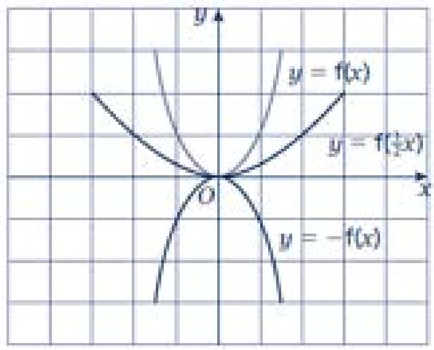
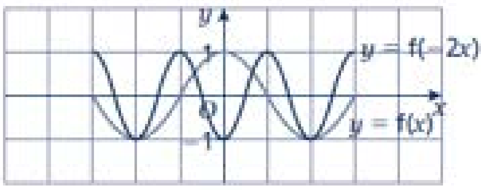


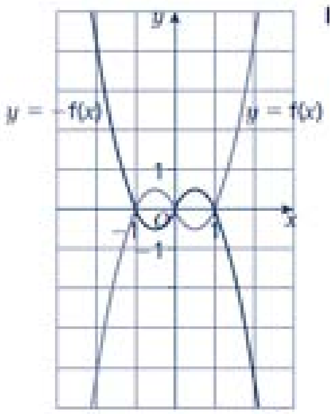
10



11 $y = \mathrm{f}(2x)$

12 $y = -2\mathrm{f}(2x)$ or $y = 2\mathrm{f}(-2x)$

13 **a**



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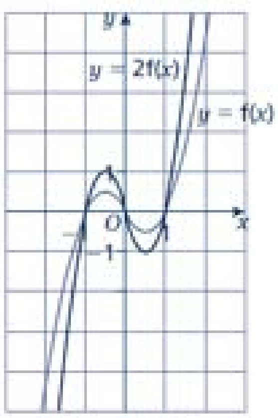

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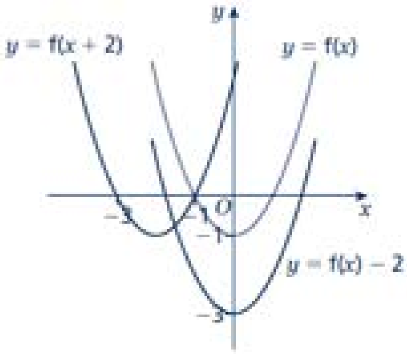

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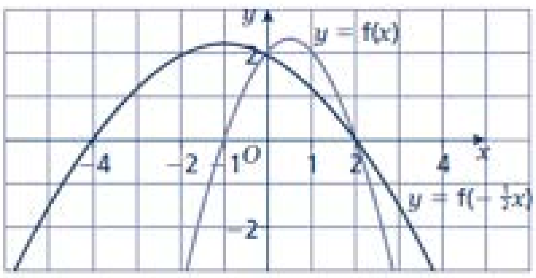
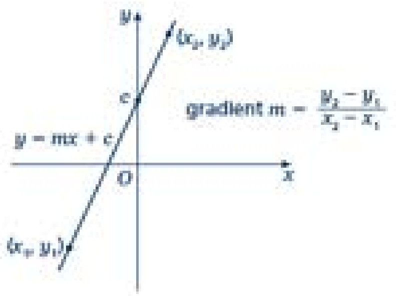


Straight line graphs

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points



- A straight line has the equation $y = mx + c$, where m is the gradient and c is the y-intercept (where $x = 0$).
- The equation of a straight line can be written in the form $ax + by + c = 0$, where a, b and c are integers.
- When given the coordinates (x_1, y_1) and (x_2, y_2) of two points on a line the gradient is calculated using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

Examples

Example 1 A straight line has gradient $-\frac{1}{2}$ and y-intercept 3.

Write the equation of the line in the form $ax + by + c = 0$.

$m = -\frac{1}{2}$ and $c = 3$ So $y = -\frac{1}{2}x + 3$ $\frac{1}{2}x + y - 3 = 0$ $x + 2y - 6 = 0$	**1** A straight line has equation $y = mx + c$. Substitute the gradient and y-intercept given in the question into this equation. **2** Rearrange the equation so all the terms are on one side and 0 is on the other side. **3** Multiply both sides by 2 to eliminate the denominator.

Example 2 Find the gradient and the y-intercept of the line with the equation $3y - 2x + 4 = 0$.

$3y - 2x + 4 = 0$ $3y = 2x - 4$ $y = \frac{2}{3}x - \frac{4}{3}$ Gradient $= m = \frac{2}{3}$ y-intercept $= c = -\frac{4}{3}$	**1** Make y the subject of the equation. **2** Divide all the terms by three to get the equation in the form $y = \ldots$ **3** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Example 3 Find the equation of the line which passes through the point (5, 13) and has gradient 3.

$m = 3$ $y = 3x + c$	**1** Substitute the gradient given in the question into the equation of a straight line $y = mx + c$.
$13 = 3 \times 5 + c$	**2** Substitute the coordinates $x = 5$ and $y = 13$ into the equation.
$13 = 15 + c$ $c = -2$	**3** Simplify and solve the equation.
$y = 3x - 2$	**4** Substitute $c = -2$ into the equation $y = 3x + c$

Example 4 Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

$x_1 = 2,\ x_2 = 8,\ y_1 = 4$ and $y_2 = 7$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-4}{8-2} = \frac{3}{6} = \frac{1}{2}$	**1** Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.
$y = \frac{1}{2}x + c$	**2** Substitute the gradient into the equation of a straight line $y = mx + c$.
$4 = \frac{1}{2} \times 2 + c$	**3** Substitute the coordinates of either point into the equation.
$c = 3$	**4** Simplify and solve the equation.
$y = \frac{1}{2}x + 3$	**5** Substitute $c = 3$ into the equation $y = \frac{1}{2}x + c$

Practice

1 Find the gradient and the y-intercept of the following equations.

a $y = 3x + 5$ **b** $y = -\frac{1}{2}x - 7$

c $2y = 4x - 3$ **d** $x + y = 5$

e $2x - 3y - 7 = 0$ **f** $5x + y - 4 = 0$

> **Hint**
> Rearrange the equations to the form $y = mx + c$

2 Copy and complete the table, giving the equation of the line in the form $y = mx + c$.

Gradient	y-intercept	Equation of the line
5	0	
–3	2	
4	–7	

3 Find, in the form $ax + by + c = 0$ where a, b and c are integers, an equation for each of the lines with the following gradients and y-intercepts.

a gradient $-\frac{1}{2}$, y-intercept -7 **b** gradient 2, y-intercept 0

c gradient $\frac{2}{3}$, y-intercept 4 **d** gradient -1.2, y-intercept -2

4 Write an equation for the line which passes though the point (2, 5) and has gradient 4.

5 Write an equation for the line which passes through the point (6, 3) and has gradient $-\frac{2}{3}$

6 Write an equation for the line passing through each of the following pairs of points.

a (4, 5), (10, 17) **b** (0, 6), (–4, 8)

c (–1, –7), (5, 23) **d** (3, 10), (4, 7)

Extend

7 The equation of a line is $2y + 3x - 6 = 0$.
Write as much information as possible about this line.

Answers

1 **a** $m = 3, c = 5$ **b** $m = -\frac{1}{2}, c = -7$

c $m = 2, c = -\frac{3}{2}$ **d** $m = -1, c = 5$

e $m = \frac{2}{3}, c = -\frac{7}{3}$ or $-2\frac{1}{3}$ **f** $m = -5, c = 4$

2

Gradient	*y*-intercept	Equation of the line
5	0	$y = 5x$
–3	2	$y = -3x + 2$
4	–7	$y = 4x - 7$

3 **a** $x + 2y + 14 = 0$ **b** $2x - y = 0$

c $2x - 3y + 12 = 0$ **d** $6x + 5y + 10 = 0$

4 $y = 4x - 3$

5 $y = -\frac{2}{3}x + 7$

6 **a** $y = 2x - 3$ **b** $y = -\frac{1}{2}x + 6$

c $y = 5x - 2$ **d** $y = -3x + 19$

7 $y = -\frac{3}{2}x + 3$, the gradient is $-\frac{3}{2}$ and the *y*-intercept is 3.

The line intercepts the axes at (0, 3) and (2, 0).

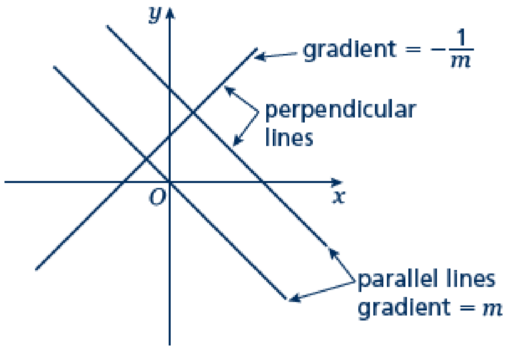
Students may sketch the line or give coordinates that lie on the line such as $\left(1, \frac{3}{2}\right)$ or $(4, -3)$.

Parallel and perpendicular lines

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- When lines are parallel they have the same gradient.
- A line perpendicular to the line with equation $y = mx + c$ has gradient $-\frac{1}{m}$.



Examples

Example 1 Find the equation of the line parallel to $y = 2x + 4$ which passes through the point (4, 9).

$y = 2x + 4$ $m = 2$ $y = 2x + c$	**1** As the lines are parallel they have the same gradient. **2** Substitute $m = 2$ into the equation of a straight line $y = mx + c$.
$9 = 2 \times 4 + c$	**3** Substitute the coordinates into the equation $y = 2x + c$
$9 = 8 + c$ $c = 1$ $y = 2x + 1$	**4** Simplify and solve the equation. **5** Substitute $c = 1$ into the equation $y = 2x + c$

Example 2 Find the equation of the line perpendicular to $y = 2x - 3$ which passes through the point $(-2, 5)$.

$y = 2x - 3$ $m = 2$ $-\frac{1}{m} = -\frac{1}{2}$	**1** As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$.
$y = -\frac{1}{2}x + c$	**2** Substitute $m = -\frac{1}{2}$ into $y = mx + c$.
$5 = -\frac{1}{2} \times (-2) + c$	**3** Substitute the coordinates (–2, 5) into the equation $y = -\frac{1}{2}x + c$
$5 = 1 + c$ $c = 4$	**4** Simplify and solve the equation.
$y = -\frac{1}{2}x + 4$	**5** Substitute $c = 4$ into $y = -\frac{1}{2}x + c$.

Example 3 A line passes through the points (0, 5) and $(9, -1)$.
Find the equation of the line which is perpendicular to the line and passes through its midpoint.

$x_1 = 0$, $x_2 = 9$, $y_1 = 5$ and $y_2 = -1$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-5}{9-0}$ $= \frac{-6}{9} = -\frac{2}{3}$	**1** Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.
$-\frac{1}{m} = \frac{3}{2}$	**2** As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$.
$y = \frac{3}{2}x + c$	**3** Substitute the gradient into the equation $y = mx + c$.
Midpoint = $\left(\frac{0+9}{2}, \frac{5+(-1)}{2}\right) = \left(\frac{9}{2}, 2\right)$	**4** Work out the coordinates of the midpoint of the line.
$2 = \frac{3}{2} \times \frac{9}{2} + c$	**5** Substitute the coordinates of the midpoint into the equation.
$c = -\frac{19}{4}$	**6** Simplify and solve the equation.
$y = \frac{3}{2}x - \frac{19}{4}$	**7** Substitute $c = -\frac{19}{4}$ into the equation $y = \frac{3}{2}x + c$.

Practice

1 Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

a $y = 3x + 1$ (3, 2) **b** $y = 3 - 2x$ (1, 3)

c $2x + 4y + 3 = 0$ (6, –3) **d** $2y - 3x + 2 = 0$ (8, 20)

2 Find the equation of the line perpendicular to $y = \frac{1}{2}x - 3$ which passes through the point (–5, 3).

> **Hint**
> If $m = \frac{a}{b}$ then the negative reciprocal $-\frac{1}{m} = -\frac{b}{a}$

3 Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.

a $y = 2x - 6$ (4, 0) **b** $y = -\frac{1}{3}x + \frac{1}{2}$ (2, 13)

c $x - 4y - 4 = 0$ (5, 15) **d** $5y + 2x - 5 = 0$ (6, 7)

4 In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

a (4, 3), (–2, –9) **b** (0, 3), (–10, 8)

Extend

5 Work out whether these pairs of lines are parallel, perpendicular or neither.

a $y = 2x + 3$
$y = 2x - 7$

b $y = 3x$
$2x + y - 3 = 0$

c $y = 4x - 3$
$4y + x = 2$

d $3x - y + 5 = 0$
$x + 3y = 1$

e $2x + 5y - 1 = 0$
$y = 2x + 7$

f $2x - y = 6$
$6x - 3y + 3 = 0$

6 The straight line $\mathbf{L_1}$ passes through the points A and B with coordinates $(-4, 4)$ and $(2, 1)$, respectively.

a Find the equation of $\mathbf{L_1}$ in the form $ax + by + c = 0$

The line $\mathbf{L_2}$ is parallel to the line $\mathbf{L_1}$ and passes through the point C with coordinates $(-8, 3)$.

b Find the equation of $\mathbf{L_2}$ in the form $ax + by + c = 0$

The line $\mathbf{L_3}$ is perpendicular to the line $\mathbf{L_1}$ and passes through the origin.

c Find an equation of $\mathbf{L_3}$

Answers

1 **a** $y = 3x - 7$ **b** $y = -2x + 5$

c $y = -\frac{1}{2}x$ **d** $y = \frac{3}{2}x + 8$

2 $y = -2x - 7$

3 **a** $y = -\frac{1}{2}x + 2$ **b** $y = 3x + 7$

c $y = -4x + 35$ **d** $y = \frac{5}{2}x - 8$

4 **a** $y = -\frac{1}{2}x$ **b** $y = 2x$

5 **a** Parallel **b** Neither **c** Perpendicular

d Perpendicular **e** Neither **f** Parallel

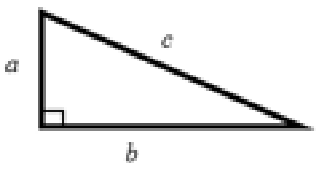
6 **a** $x + 2y - 4 = 0$ **b** $x + 2y + 2 = 0$ **c** $y = 2x$

Pythagoras' theorem

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

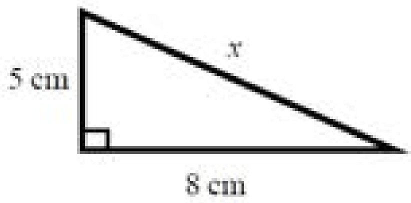
Key points

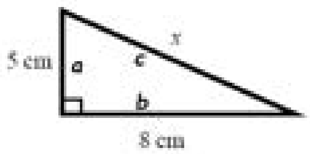
- In a right-angled triangle the longest side is called the hypotenuse.
- Pythagoras' theorem states that for a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.
  $c^2 = a^2 + b^2$



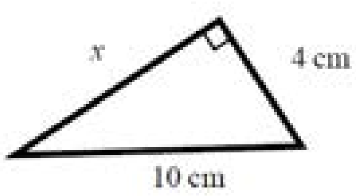
Examples

Example 1 Calculate the length of the hypotenuse.
Give your answer to 3 significant figures.



$c^2 = a^2 + b^2$  	**1** Always start by stating the formula for Pythagoras' theorem and labelling the hypotenuse *c* and the other two sides *a* and *b*.
$x^2 = 5^2 + 8^2$ $x^2 = 25 + 64$ $x^2 = 89$ $x = \sqrt{89}$ $x = 9.433\ 981\ 13...$ $x = 9.43$ cm	**2** Substitute the values of *a*, *b* and *c* into the formula for Pythagoras' theorem. **3** Use a calculator to find the square root. **4** Round your answer to 3 significant figures and write the units with your answer.

Example 2 Calculate the length x.
Give your answer in surd form.

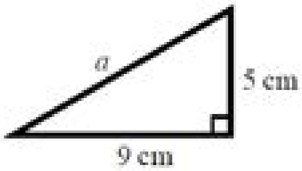


$c^2 = a^2 + b^2$ $10^2 = x^2 + 4^2$ $100 = x^2 + 16$ $x^2 = 84$ $x = \sqrt{84}$ $x = 2\sqrt{21}$ cm	**1** Always start by stating the formula for Pythagoras' theorem. **2** Substitute the values of a, b and c into the formula for Pythagoras' theorem. **3** Simplify the surd where possible and write the units in your answer.

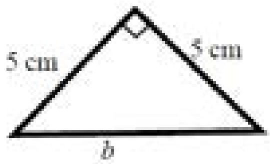
Practice

1 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

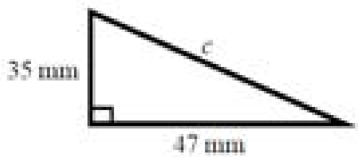
a



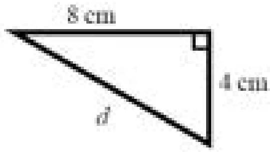
b



c

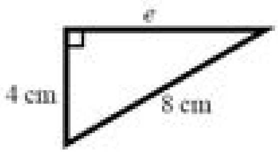


d

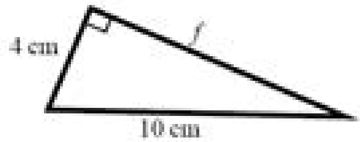


2 Work out the length of the unknown side in each triangle.
Give your answers in surd form.

a

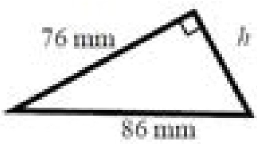


b



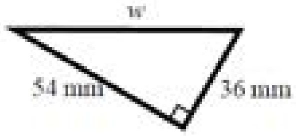
c

d

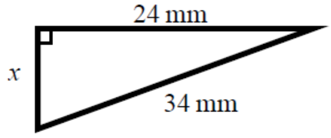


3 Work out the length of the unknown side in each triangle.
Give your answers in surd form.

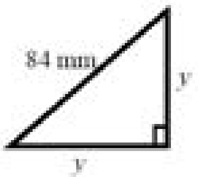
a



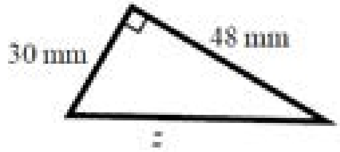
b



c



d



4 A rectangle has length 84 mm and width 45 mm.
Calculate the length of the diagonal of the rectangle.
Give your answer correct to 3 significant figures.

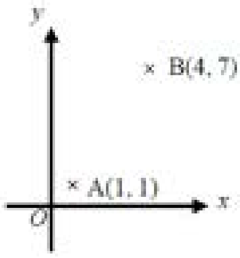
> **Hint**
>
> Draw a sketch of the rectangle.

Extend

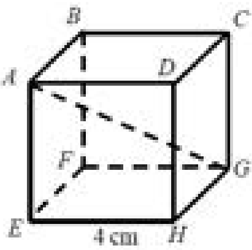
5 A yacht is 40 km due North of a lighthouse.
A rescue boat is 50 km due East of the same lighthouse.
Work out the distance between the yacht and the rescue boat.
Give your answer correct to 3 significant figures.

> **Hint**
>
> Draw a diagram using the information given in the question.

6 Points A and B are shown on the diagram.
Work out the length of the line AB.
Give your answer in surd form.



7 A cube has length 4 cm.
Work out the length of the diagonal *AG*.
Give your answer in surd form.



Answers

1	**a**	10.3 cm	**b**	7.07 cm
	c	58.6 mm	**d**	8.94 cm
2	**a**	$4\sqrt{3}$ cm	**b**	$2\sqrt{21}$ cm
	c	$8\sqrt{17}$ mm	**d**	$18\sqrt{5}$ mm
3	**a**	$18\sqrt{13}$ mm	**b**	$2\sqrt{145}$ mm
	c	$42\sqrt{2}$ mm	**d**	$6\sqrt{89}$ mm

4 95.3 mm

5 64.0 km

6 $3\sqrt{5}$ units

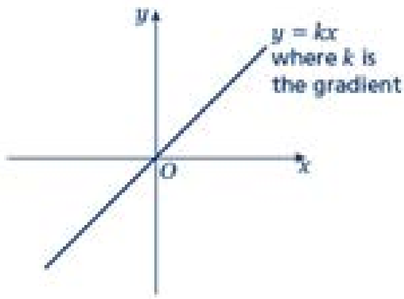
7 $4\sqrt{3}$ cm

Proportion

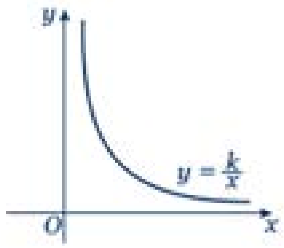
A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- Two quantities are in direct proportion when, as one quantity increases, the other increases at the same rate. Their ratio remains the same.
- 'y is directly proportional to x' is written as $y \propto x$. If $y \propto x$ then $y = kx$, where k is a constant.
- When x is directly proportional to y, the graph is a straight line passing through the origin.



- Two quantities are in inverse proportion when, as one quantity increases, the other decreases at the same rate.
- 'y is inversely proportional to x' is written as $y \propto \frac{1}{x}$. If $y \propto \frac{1}{x}$ then $y = \frac{k}{x}$, where k is a constant.
- When x is inversely proportional to y the graph is the same shape as the graph of $y = \frac{1}{x}$



Examples

Example 1 y is directly proportional to x.
When $y = 16$, $x = 5$.
a Find x when $y = 30$.
b Sketch the graph of the formula.

a $y \propto x$	**1** Write y is directly proportional to x, using the symbol $\propto$.
$y = kx$	**2** Write the equation using k.
$16 = k \times 5$	**3** Substitute $y = 16$ and $x = 5$ into $y = kx$.
$k = 3.2$	**4** Solve the equation to find k.
$y = 3.2x$	**5** Substitute the value of k back into the equation $y = kx$.
When $y = 30$, $30 = 3.2 \times x$ $x = 9.375$	**6** Substitute $y = 30$ into $y = 3.2x$ and solve to find x when $y = 30$.

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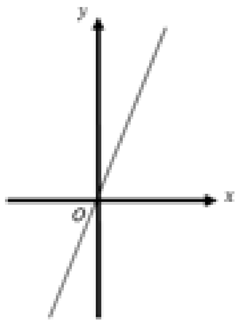
b 	**7** The graph of $y = 3.2x$ is a straight line passing through (0, 0) with a gradient of 3.2.

Example 2 y is directly proportional to x^2.
When $x = 3$, $y = 45$.
a Find y when $x = 5$.
b Find x when $y = 20$.

a $y \propto x^2$	**1** Write y is directly proportional to x^2, using the symbol $\propto$.
$y = kx^2$ $45 = k \times 3^2$	**2** Write the equation using k. **3** Substitute $y = 45$ and $x = 3$ into $y = kx^2$.
$k = 5$ $y = 5x^2$	**4** Solve the equation to find k. **5** Substitute the value of k back into the equation $y = kx^2$.
When $x = 5$, $y = 5 \times 5^2$ $y = 125$	**6** Substitute $x = 5$ into $y = 5x^2$ and solve to find y when $x = 5$.
b $20 = 5 \times x^2$ $x^2 = 4$ $x = \pm 2$	**7** Substitute $y = 20$ into $y = 5x^2$ and solve to find x when $y = 4$.

Example 3 P is inversely proportional to Q.
When $P = 100$, $Q = 10$.
Find Q when $P = 20$.

$P \propto \frac{1}{Q}$	**1** Write P is inversely proportional to Q, using the symbol $\propto$.
$P = \frac{k}{Q}$	**2** Write the equation using k.
$100 = \frac{k}{10}$	**3** Substitute $P = 100$ and $Q = 10$.
$k = 1000$	**4** Solve the equation to find k.
$P = \frac{1000}{Q}$	**5** Substitute the value of k into $P = \frac{k}{Q}$
$20 = \frac{1000}{Q}$ $Q = \frac{1000}{20} = 50$	**6** Substitute $P = 20$ into $P = \frac{1000}{Q}$ and solve to find Q when $P = 20$.

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Practice

> **Hint**
>
> Substitute the values given for P and h into the formula to calculate k.

1 Paul gets paid an hourly rate. The amount of pay (£P) is directly proportional to the number of hours (h) he works.
When he works 8 hours he is paid £56.
If Paul works for 11 hours, how much is he paid?

2 x is directly proportional to y.
$x = 35$ when $y = 5$.

- **a** Find a formula for x in terms of y.
- **b** Sketch the graph of the formula.
- **c** Find x when $y = 13$.
- **d** Find y when $x = 63$.

3 Q is directly proportional to the square of Z.
$Q = 48$ when $Z = 4$.

- **a** Find a formula for Q in terms of Z.
- **b** Sketch the graph of the formula.
- **c** Find Q when $Z = 5$.
- **d** Find Z when $Q = 300$.

4 y is directly proportional to the square of x.
$x = 2$ when $y = 10$.

- **a** Find a formula for y in terms of x.
- **b** Sketch the graph of the formula.
- **c** Find x when $y = 90$.

5 B is directly proportional to the square root of C.
$C = 25$ when $B = 10$.

- **a** Find B when $C = 64$.
- **b** Find C when $B = 20$.

6 C is directly proportional to D.
$C = 100$ when $D = 150$.
Find C when $D = 450$.

7 y is directly proportional to x.
$x = 27$ when $y = 9$.
Find x when $y = 3.7$.

8 m is proportional to the cube of n.
$m = 54$ when $n = 3$.
Find n when $m = 250$.

Extend

9 s is inversely proportional to t.

a Given that $s = 2$ when $t = 2$, find a formula for s in terms of t.

b Sketch the graph of the formula.

c Find t when $s = 1$.

10 a is inversely proportional to b.
$a = 5$ when $b = 20$.

a Find a when $b = 50$.

b Find b when $a = 10$.

11 v is inversely proportional to w.
$w = 4$ when $v = 20$.

a Find a formula for v in terms of w.

b Sketch the graph of the formula.

c Find w when $v = 2$.

12 L is inversely proportional to W.
$L = 12$ when $W = 3$.
Find W when $L = 6$.

13 s is inversely proportional to t.
$s = 6$ when $t = 12$.

a Find s when $t = 3$.

b Find t when $s = 18$.

14 y is inversely proportional to x^2.
$y = 4$ when $x = 2$.
Find y when $x = 4$.

15 y is inversely proportional to the square root of x.
$x = 25$ when $y = 1$.
Find x when $y = 5$.

16 a is inversely proportional to b.
$a = 0.05$ when $b = 4$.

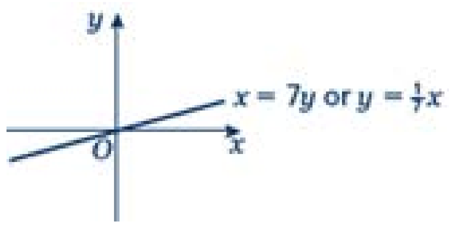
a Find a when $b = 2$.

b Find b when $a = 2$.

Answers

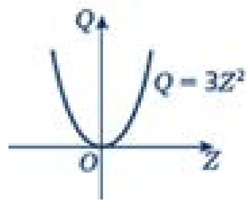
1 £77

2 **a** $x = 7y$ **b**



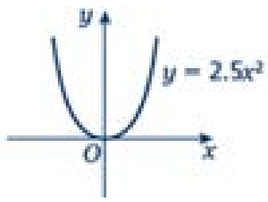
c 91 **d** 9

3 **a** $Q = 3Z^2$ **b**



c 75 **d** ±10

4 **a** $y = 2.5x^2$ **b**



c ±6

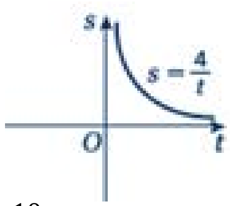
5 **a** 16 **b** 100

6 300

7 11.1

8 5

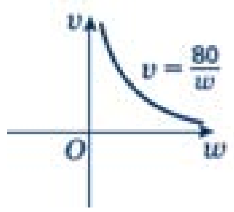
9 **a** $s = \dfrac{4}{t}$ **b**



c 4

10 **a** 2 **b** 10

11 **a** $v = \dfrac{80}{w}$ **b**



c 40

12 6

13 **a** 24 **b** 4

14 1

15 1

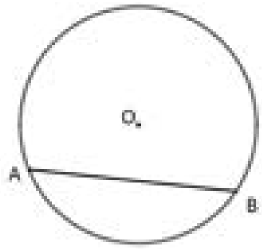
16 **a** 0.1 **b** 0.1

Circle theorems

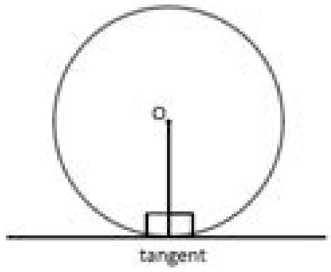
A LEVEL LINKS
Scheme of work: 2b. Circles – equation of a circle, geometric problems on a grid

Key points

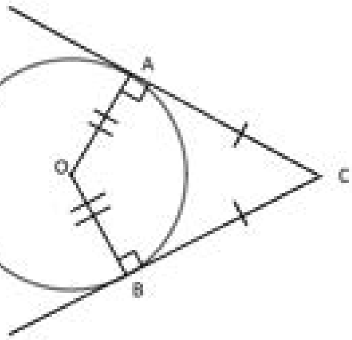
A chord is a straight line joining two points on the circumference of a circle.
So AB is a chord.



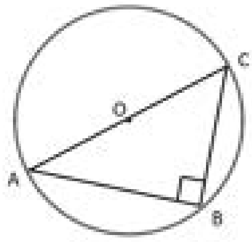
A tangent is a straight line that touches the circumference of a circle at only one point.
The angle between a tangent and the radius is 90°.



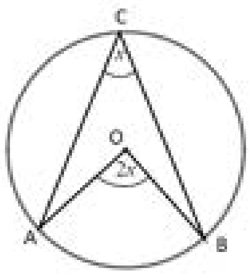
Two tangents on a circle that meet at a point outside the circle are equal in length.
So AC = BC.



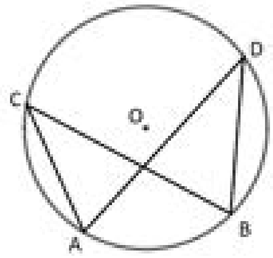
The angle in a semicircle is a right angle.
So angle ABC = 90°.



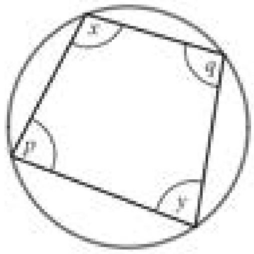
When two angles are subtended by the same arc, the angle at the centre of a circle is twice the angle at the circumference.
So angle AOB = 2 × angle ACB.



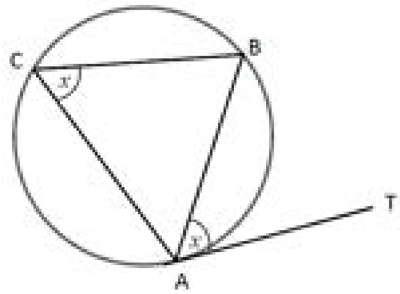
Angles subtended by the same arc at the circumference are equal. This means that angles in the same segment are equal.
So angle ACB = angle ADB and
angle CAD = angle CBD.



A cyclic quadrilateral is a quadrilateral with all four vertices on the circumference of a circle.
Opposite angles in a cyclic quadrilateral total 180°.
So $x + y = 180°$ and $p + q = 180°$.

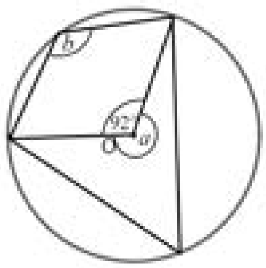


The angle between a tangent and chord is equal to the angle in the alternate segment, this is known as the alternate segment theorem.
So angle BAT = angle ACB.



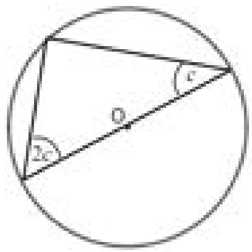
Examples

Example 1 Work out the size of each angle marked with a letter.
Give reasons for your answers.



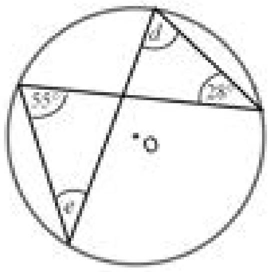
Angle $a = 360° - 92°$ $= 268°$ as the angles in a full turn total 360°.	**1** The angles in a full turn total 360°.
Angle $b = 268° \div 2$ $= 134°$ as when two angles are subtended by the same arc, the angle at the centre of a circle is twice the angle at the circumference.	**2** Angles a and b are subtended by the same arc, so angle b is half of angle a.

Example 2 Work out the size of the angles in the triangle.
Give reasons for your answers.



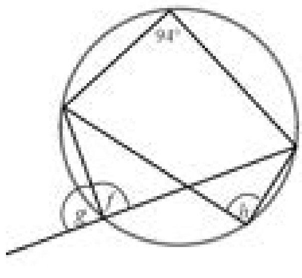
Angles are 90°, $2c$ and c. $90° + 2c + c = 180°$ $90° + 3c = 180°$ $3c = 90°$ $c = 30°$ $2c = 60°$ The angles are 30°, 60° and 90° as the angle in a semi-circle is a right angle and the angles in a triangle total 180°.	**1** The angle in a semicircle is a right angle. **2** Angles in a triangle total 180°. **3** Simplify and solve the equation.

Example 3 Work out the size of each angle marked with a letter.
Give reasons for your answers.



Angle d = 55° as angles subtended by the same arc are equal. Angle e = 28° as angles subtended by the same arc are equal.	**1** Angles subtended by the same arc are equal so angle 55° and angle d are equal. **2** Angles subtended by the same arc are equal so angle 28° and angle e are equal.

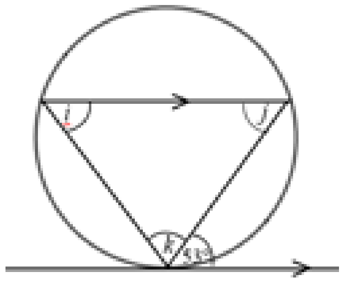
Example 4 Work out the size of each angle marked with a letter.
Give reasons for your answers.



Angle $f = 180° - 94°$ $= 86°$ as opposite angles in a cyclic quadrilateral total 180°.	**1** Opposite angles in a cyclic quadrilateral total 180° so angle 94° and angle f total 180°. *(continued on next page)*

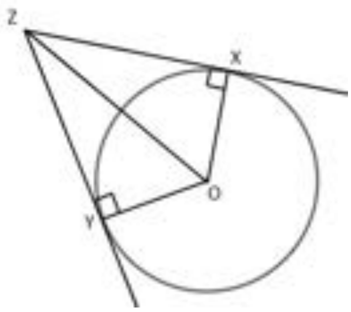
Angle $g = 180° - 86°$ $= 84°$ as angles on a straight line total 180°. Angle h = angle f = 86° as angles subtended by the same arc are equal.	**2** Angles on a straight line total 180° so angle f and angle g total 180°. **3** Angles subtended by the same arc are equal so angle f and angle h are equal.

Example 5 Work out the size of each angle marked with a letter.
Give reasons for your answers.



Angle i = 53° because of the alternate segment theorem. Angle j = 53° because it is the alternate angle to 53°. Angle $k = 180° - 53° - 53°$ $= 74°$ as angles in a triangle total 180°.	**1** The angle between a tangent and chord is equal to the angle in the alternate segment. **2** As there are two parallel lines, angle 53° is equal to angle j because they are alternate angles. **3** The angles in a triangle total 180°, so $i + j + k = 180°$.

Example 6 XZ and YZ are two tangents to a circle with centre O.
Prove that triangles XZO and YZO are congruent.

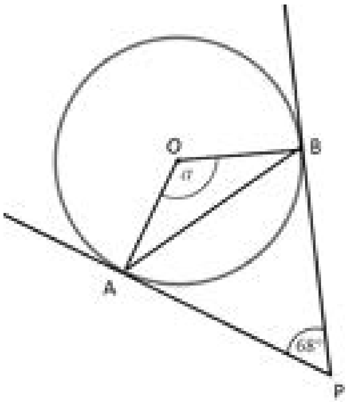


Angle OXZ = 90° and angle OYZ = 90° as the angles in a semicircle are right angles. OZ is a common line and is the hypotenuse in both triangles. OX = OY as they are radii of the same circle. So triangles XZO and YZO are congruent, RHS.	For two triangles to be congruent you need to show one of the following. • All three corresponding sides are equal (SSS). • Two corresponding sides and the included angle are equal (SAS). • One side and two corresponding angles are equal (ASA). • A right angle, hypotenuse and a shorter side are equal (RHS).

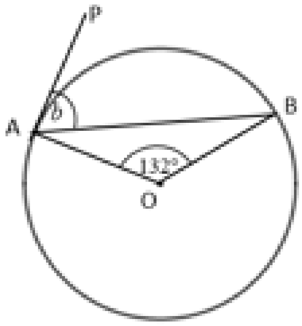
Practice

1 Work out the size of each angle marked with a letter.
Give reasons for your answers.

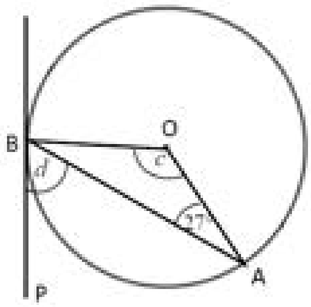
a



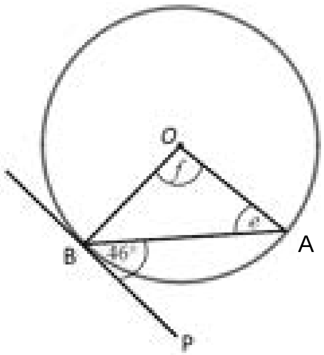
b



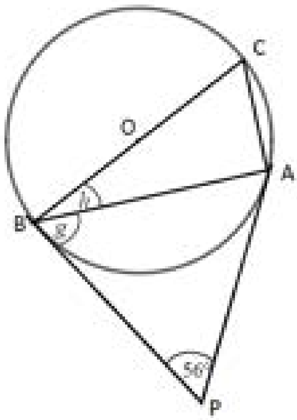
c



d

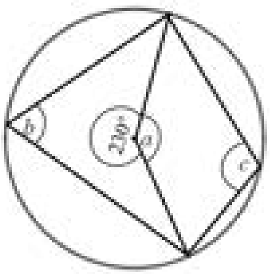


e



2 Work out the size of each angle marked with a letter.
Give reasons for your answers.

a



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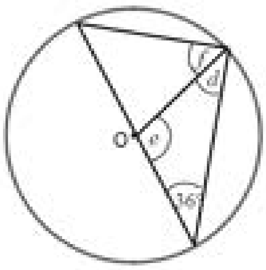

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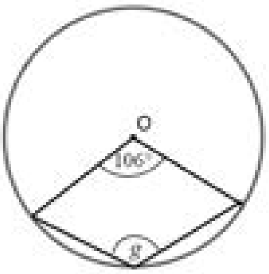
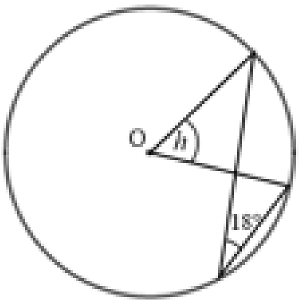


> **Hint**
>
> The reflex angle at point O and angle *g* are subtended by the same arc. So the reflex angle is twice the size of angle *g*.

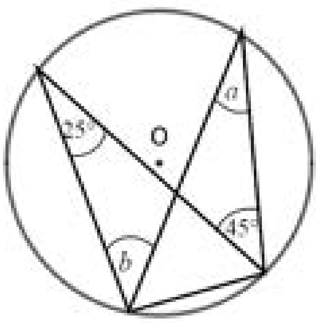
d



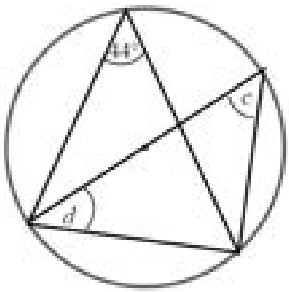
> **Hint**
>
> Angle 18° and angle *h* are subtended by the same arc.

3 Work out the size of each angle marked with a letter.
Give reasons for your answers.

a

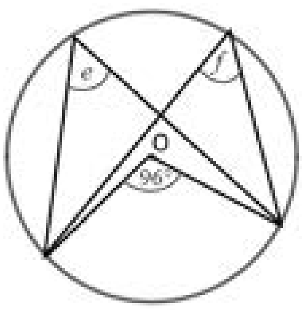


b

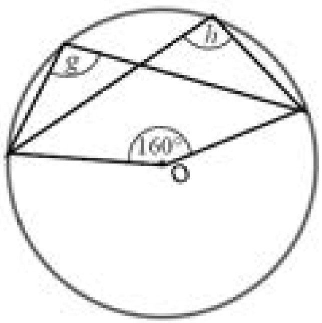


> **Hint**
>
> One of the angles is in a semicircle.

c

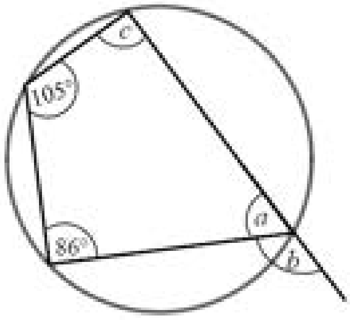


d



4 Work out the size of each angle marked with a letter.
Give reasons for your answers.

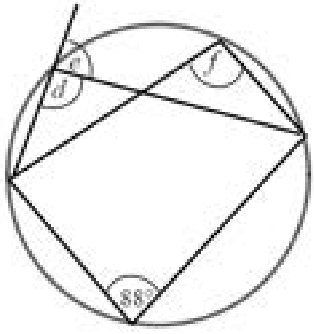
a



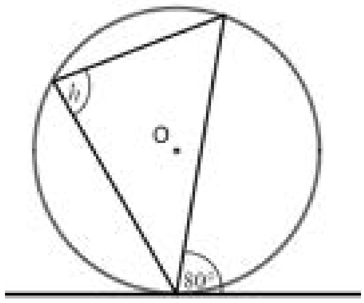
Hint

An exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.

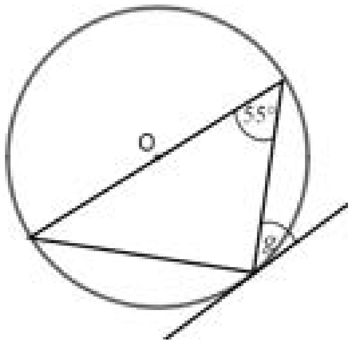
b



c



d



Hint

One of the angles is in a semicircle.

Extend

5 Prove the alternate segment theorem.

Answers

1 **a** $a = 112°$, angle OAP = angle OBP = 90° and angles in a quadrilateral total 360°.

b $b = 66°$, triangle OAB is isosceles, Angle OAP = 90° as AP is tangent to the circle.

c $c = 126°$, triangle OAB is isosceles.
$d = 63°$, Angle OBP = 90° as BP is tangent to the circle.

d $e = 44°$, the triangle is isosceles, so angles *e* and angle OBA are equal. The angle OBP = 90° as BP is tangent to the circle.
$f = 92°$, the triangle is isosceles.

e $g = 62°$, triangle ABP is isosceles as AP and BP are both tangents to the circle.
$h = 28°$, the angle OBP = 90°.

2 **a** $a = 130°$, angles in a full turn total 360°.
$b = 65°$, the angle at the centre of a circle is twice the angle at the circumference.
$c = 115°$, opposite angles in a cyclic quadrilateral total 180°.

b $d = 36°$, isosceles triangle.
$e = 108°$, angles in a triangle total 180°.
$f = 54°$, angle in a semicircle is 90°.

c $g = 127°$, angles at a full turn total 360°, the angle at the centre of a circle is twice the angle at the circumference.

d $h = 36°$, the angle at the centre of a circle is twice the angle at the circumference.

3 **a** $a = 25°$, angles in the same segment are equal.
$b = 45°$, angles in the same segment are equal.

b $c = 44°$, angles in the same segment are equal.
$d = 46°$, the angle in a semicircle is 90° and the angles in a triangle total 180°.

c $e = 48°$, the angle at the centre of a circle is twice the angle at the circumference.
$f = 48°$, angles in the same segment are equal.

d $g = 100°$, angles at a full turn total 360°, the angle at the centre of a circle is twice the angle at the circumference.
$h = 100°$, angles in the same segment are equal.

4 **a** $a = 75°$, opposite angles in a cyclic quadrilateral total 180°.
$b = 105°$, angles on a straight line total 180°.
$c = 94°$, opposite angles in a cyclic quadrilateral total 180°.

b $d = 92°$, opposite angles in a cyclic quadrilateral total 180°.
$e = 88°$, angles on a straight line total 180°.
$f = 92°$, angles in the same segment are equal.

c $h = 80°$, alternate segment theorem.

d $g = 35°$, alternate segment theorem and the angle in a semicircle is 90°.

5 Angle BAT = x.

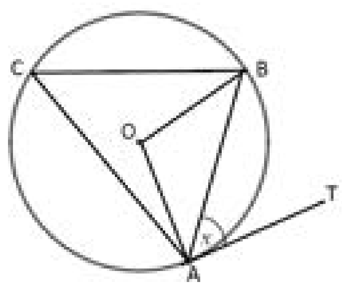
Angle OAB = 90° − x because the angle between the tangent and the radius is 90°.

OA = OB because radii are equal.

Angle OAB = angle OBA because the base of isosceles triangles are equal.

Angle AOB = 180° − (90° − x) − (90° − x) = 2x because angles in a triangle total 180°.

Angle ACB = 2x ÷ 2 = x because the angle at the centre is twice the angle at the circumference.

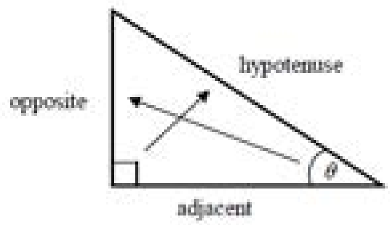


Trigonometry in right-angled triangles

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs

Key points

- In a right-angled triangle:
  - the side opposite the right angle is called the hypotenuse
  - the side opposite the angle θ is called the opposite
  - the side next to the angle θ is called the adjacent.



- In a right-angled triangle:
  - the ratio of the opposite side to the hypotenuse is the sine of angle θ, $\sin\theta = \frac{\text{opp}}{\text{hyp}}$
  - the ratio of the adjacent side to the hypotenuse is the cosine of angle θ, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$
  - the ratio of the opposite side to the adjacent side is the tangent of angle θ, $\tan\theta = \frac{\text{opp}}{\text{adj}}$

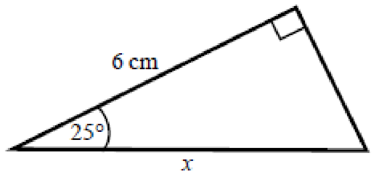
- If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$.

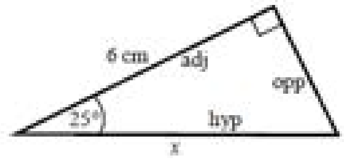
- The sine, cosine and tangent of some angles may be written exactly.

	0	**30°**	**45°**	**60°**	**90°**
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

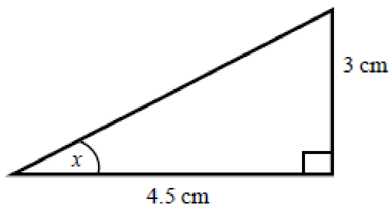
Examples

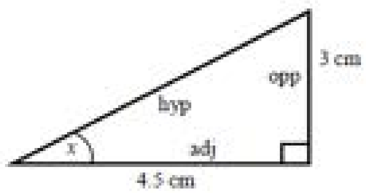
Example 1 Calculate the length of side x.
Give your answer correct to 3 significant figures.



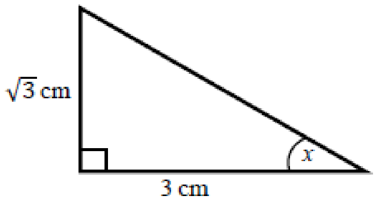
	**1** Always start by labelling the sides.
$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	**2** You are given the adjacent and the hypotenuse so use the cosine ratio.
$\cos 25° = \frac{6}{x}$	**3** Substitute the sides and angle into the cosine ratio.
$x = \frac{6}{\cos 25°}$	**4** Rearrange to make x the subject.
$x = 6.620\,267\,5...$	**5** Use your calculator to work out $6 \div \cos 25°$.
$x = 6.62$ cm	**6** Round your answer to 3 significant figures and write the units in your answer.

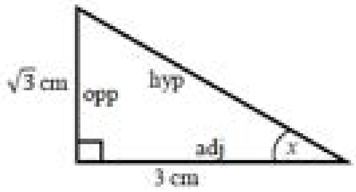
Example 2 Calculate the size of angle x.
Give your answer correct to 3 significant figures.



	**1** Always start by labelling the sides.
$\tan\theta = \frac{\text{opp}}{\text{adj}}$	**2** You are given the opposite and the adjacent so use the tangent ratio.
$\tan x = \frac{3}{4.5}$	**3** Substitute the sides and angle into the tangent ratio.
$x = \tan^{-1}\left(\frac{3}{4.5}\right)$	**4** Use $\tan^{-1}$ to find the angle.
$x = 33.690\,067\,5...$	**5** Use your calculator to work out $\tan^{-1}(3 \div 4.5)$.
$x = 33.7°$	**6** Round your answer to 3 significant figures and write the units in your answer.

Example 3 Calculate the exact size of angle x.

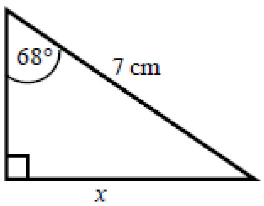


	**1** Always start by labelling the sides.
$\tan\theta = \frac{\text{opp}}{\text{adj}}$	**2** You are given the opposite and the adjacent so use the tangent ratio.
$\tan x = \frac{\sqrt{3}}{3}$	**3** Substitute the sides and angle into the tangent ratio.
$x = 30°$	**4** Use the table from the key points to find the angle.

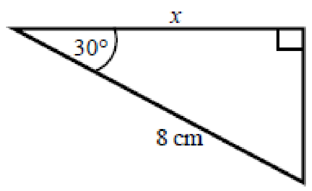
Practice

1 Calculate the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

a



b



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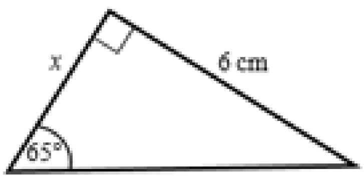
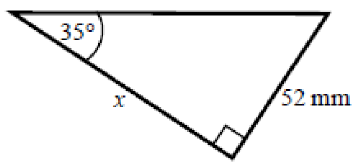
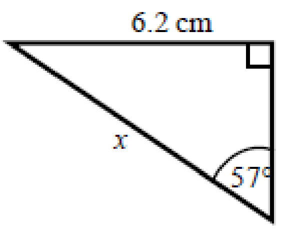


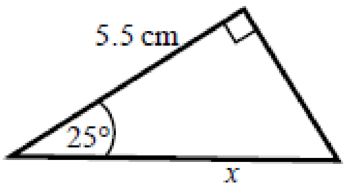
d



e

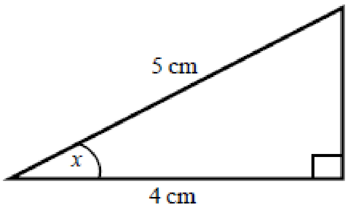


f

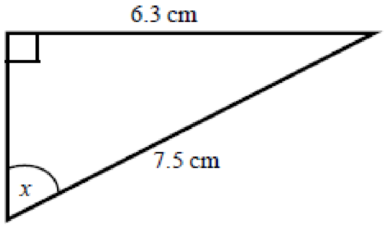


2 Calculate the size of angle x in each triangle.
Give your answers correct to 1 decimal place.

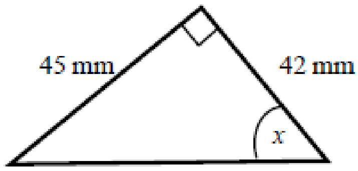
a



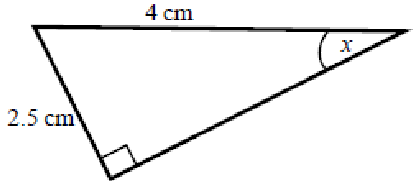
b



c

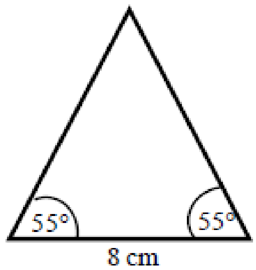


d



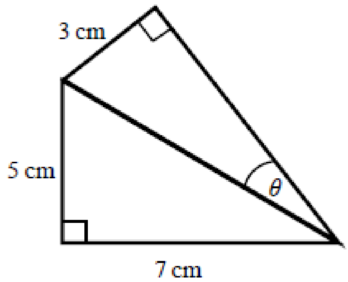
3 Work out the height of the isosceles triangle.
Give your answer correct to 3 significant figures.

> **Hint:**
>
> Split the triangle into two right-angled triangles.



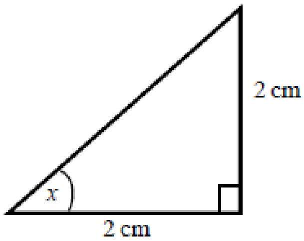
4 Calculate the size of angle θ.
Give your answer correct to 1 decimal place.

> **Hint:**
>
> First work out the length of the common side to both triangles, leaving your answer in surd form.

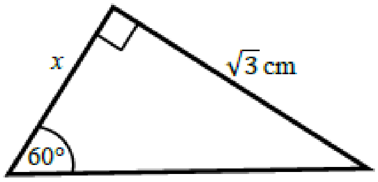


5 Find the exact value of x in each triangle.

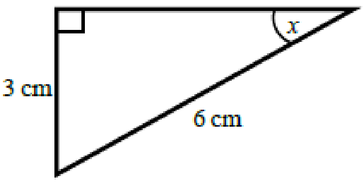
a



b



c



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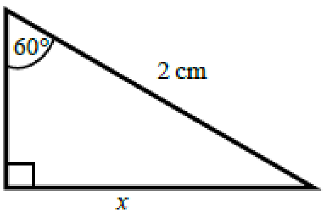
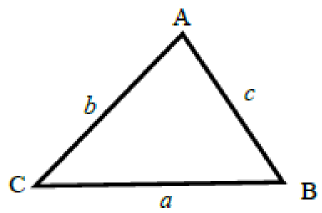


The cosine rule

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs
Textbook: Pure Year 1, 9.1 The cosine rule

Key points

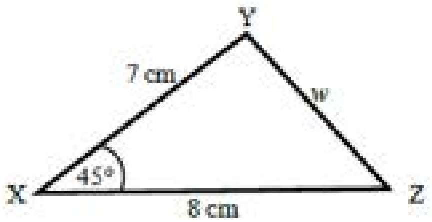
- *a* is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

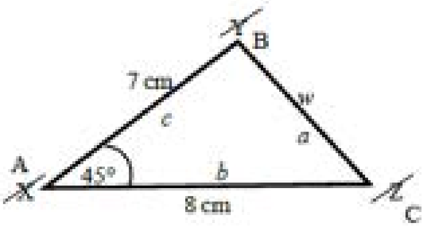


- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula $a^2 = b^2 + c^2 - 2bc\cos A$.
- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
- To calculate an unknown angle use the formula $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.

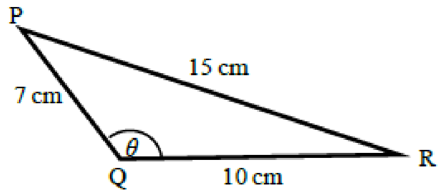
Examples

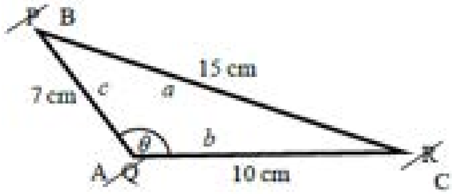
Example 4 Work out the length of side *w*.
Give your answer correct to 3 significant figures.



	**1** Always start by labelling the angles and sides.
$a^2 = b^2 + c^2 - 2bc\cos A$	**2** Write the cosine rule to find the side.
$w^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 45°$	**3** Substitute the values *a*, *b* and *A* into the formula.
$w^2 = 33.804\,040\,51...$ $w = \sqrt{33.80404051}$ $w = 5.81$ cm	**4** Use a calculator to find w^2 and then *w*. **5** Round your final answer to 3 significant figures and write the units in your answer.

Example 5 Work out the size of angle θ.
Give your answer correct to 1 decimal place.

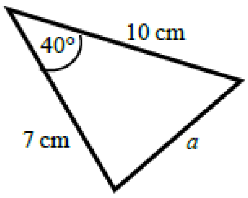


	**1** Always start by labelling the angles and sides.
$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$	**2** Write the cosine rule to find the angle.
$\cos\theta = \dfrac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$	**3** Substitute the values *a*, *b* and *c* into the formula.
$\cos\theta = \dfrac{-76}{140}$	**4** Use $\cos^{-1}$ to find the angle.
θ = 122.878 349...	**5** Use your calculator to work out $\cos^{-1}(-76 \div 140)$.
θ = 122.9°	**6** Round your answer to 1 decimal place and write the units in your answer.

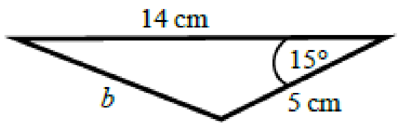
Practice

6 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

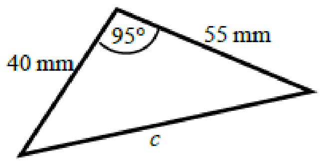
a



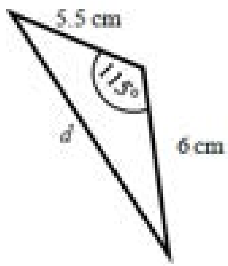
b



c

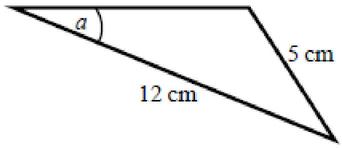


d

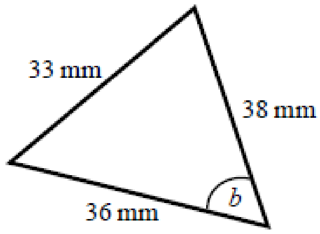


7 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

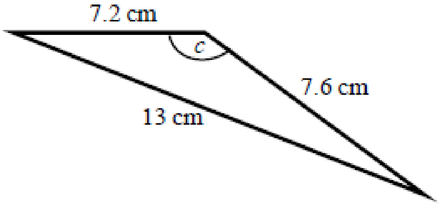
a



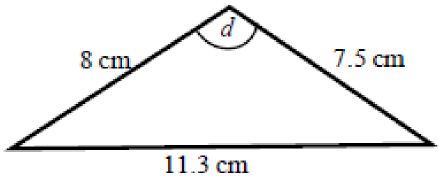
b



c

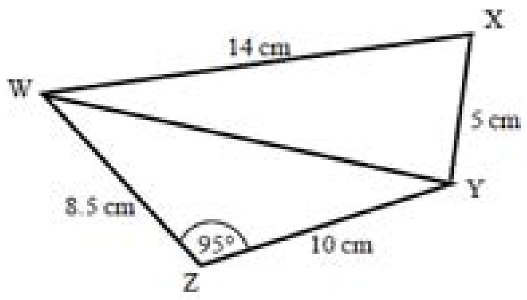


d



8 **a** Work out the length of WY.
Give your answer correct to
3 significant figures.

b Work out the size of angle WXY.
Give your answer correct to
1 decimal place.

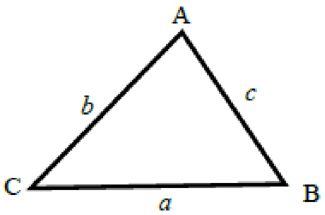


The sine rule

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs
Textbook: Pure Year 1, 9.2 The sine rule

Key points

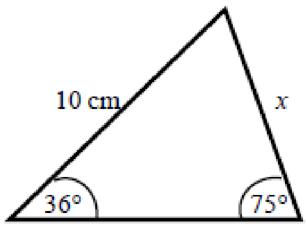
- *a* is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

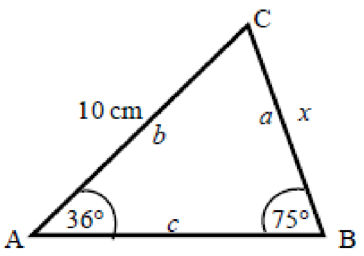


- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
- To calculate an unknown side use the formula $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
- To calculate an unknown angle use the formula $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

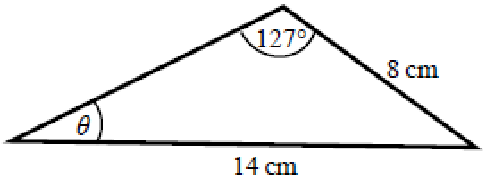
Examples

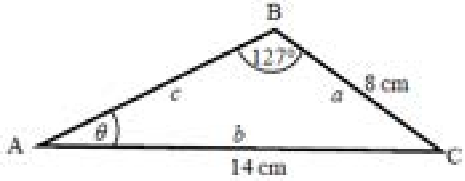
Example 6 Work out the length of side *x*.
Give your answer correct to 3 significant figures.



	**1** Always start by labelling the angles and sides.
$\frac{a}{\sin A} = \frac{b}{\sin B}$	**2** Write the sine rule to find the side.
$\frac{x}{\sin 36°} = \frac{10}{\sin 75°}$	**3** Substitute the values *a*, *b*, *A* and *B* into the formula.
$x = \frac{10 \times \sin 36°}{\sin 75°}$	**4** Rearrange to make *x* the subject.
$x = 6.09$ cm	**5** Round your answer to 3 significant figures and write the units in your answer.

Example 7 Work out the size of angle θ.
Give your answer correct to 1 decimal place.

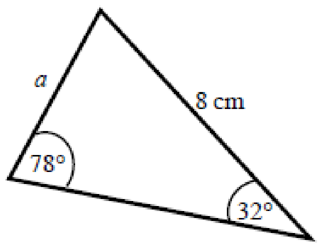


	**1** Always start by labelling the angles and sides.
$\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$	**2** Write the sine rule to find the angle.
$\dfrac{\sin\theta}{8} = \dfrac{\sin 127°}{14}$	**3** Substitute the values *a*, *b*, *A* and *B* into the formula.
$\sin\theta = \dfrac{8 \times \sin 127°}{14}$	**4** Rearrange to make sin θ the subject.
$\theta = 27.2°$	**5** Use $\sin^{-1}$ to find the angle. Round your answer to 1 decimal place and write the units in your answer.

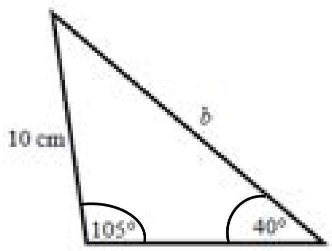
Practice

9 Find the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

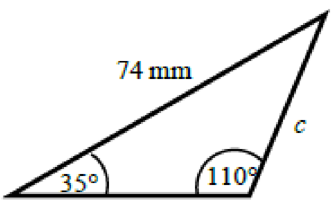
a



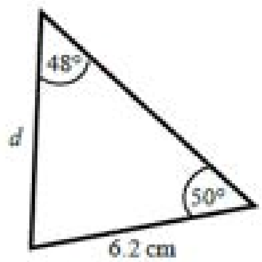
b



c

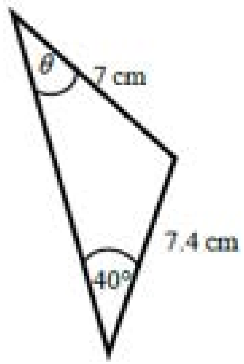


d

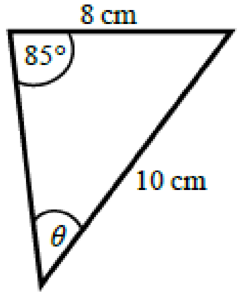


10 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

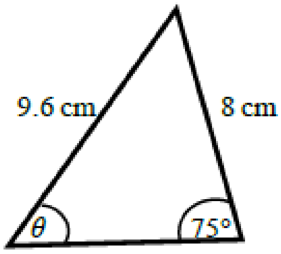
a



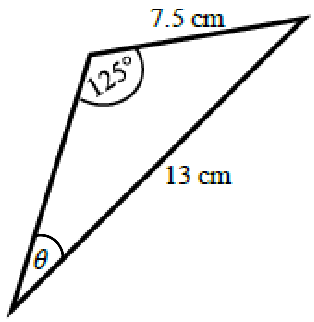
b



c

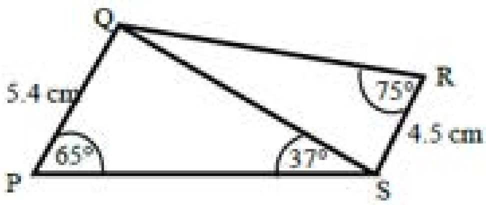


d



11 **a** Work out the length of QS.
Give your answer correct to 3 significant figures.

b Work out the size of angle RQS.
Give your answer correct to 1 decimal place.

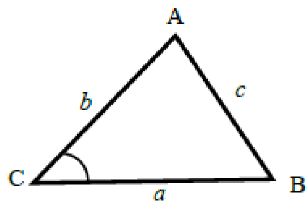


Areas of triangles

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs
Textbook: Pure Year 1, 9.3 Areas of triangles

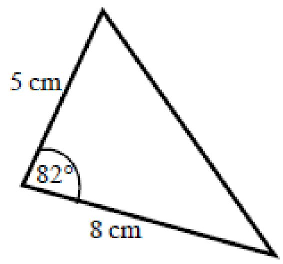
Key points

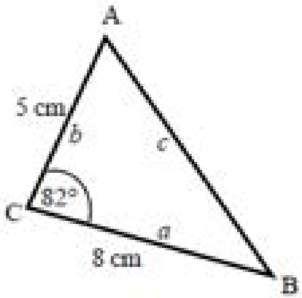
- a is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.
- The area of the triangle is $\frac{1}{2}ab\sin C$.



Examples

Example 8 Find the area of the triangle.

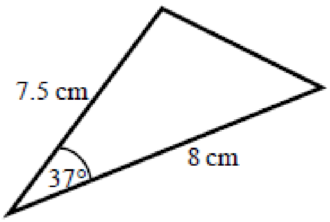


 	**1** Always start by labelling the sides and angles of the triangle.
Area = $\frac{1}{2}ab\sin C$	**2** State the formula for the area of a triangle.
Area = $\frac{1}{2}\times 8\times 5\times \sin 82°$	**3** Substitute the values of a, b and C into the formula for the area of a triangle.
Area = 19.805 361...	**4** Use a calculator to find the area.
Area = 19.8 cm^2	**5** Round your answer to 3 significant figures and write the units in your answer.

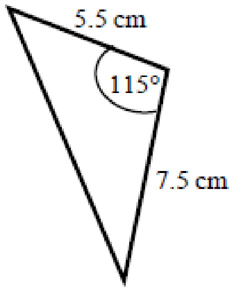
Practice

12 Work out the area of each triangle.
Give your answers correct to 3 significant figures.

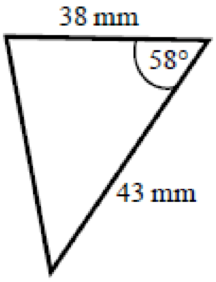
a



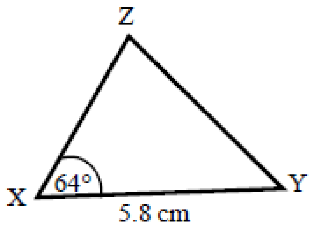
b



c



13 The area of triangle XYZ is 13.3 cm^2.
Work out the length of XZ.



Hint:

Rearrange the formula to make a side the subject.

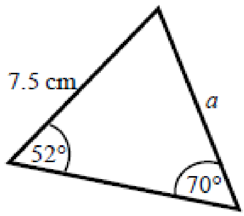
Extend

14 Find the size of each lettered angle or side.
Give your answers correct to 3 significant figures.

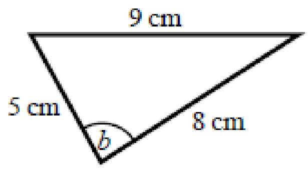
Hint:

For each one, decide whether to use the cosine or sine rule.

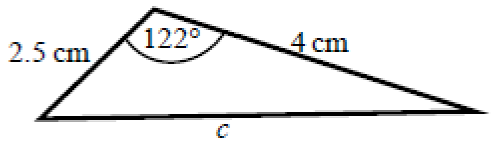
a



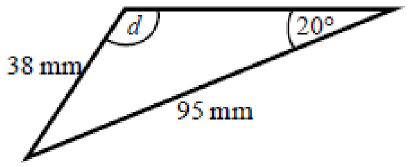
b



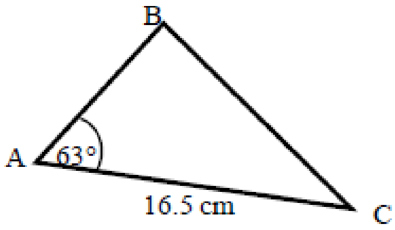
c



d



15 The area of triangle ABC is 86.7 cm^2.
Work out the length of BC.
Give your answer correct to 3 significant figures.



Answers

1	**a**	6.49 cm	**b**	6.93 cm	**c**	2.80 cm		
	d	74.3 mm	**e**	7.39 cm	**f**	6.07 cm		
2	**a**	36.9°	**b**	57.1°	**c**	47.0°	**d**	38.7°
3	5.71 cm							
4	20.4°							
5	**a**	45°	**b**	1 cm	**c**	30°	**d**	$\sqrt{3}$ cm
6	**a**	6.46 cm	**b**	9.26 cm	**c**	70.8 mm	**d**	9.70 cm
7	**a**	22.2°	**b**	52.9°	**c**	122.9°	**d**	93.6°
8	**a**	13.7 cm	**b**	76.0°				
9	**a**	4.33 cm	**b**	15.0 cm	**c**	45.2 mm	**d**	6.39 cm
10	**a**	42.8°	**b**	52.8°	**c**	53.6°	**d**	28.2°
11	**a**	8.13 cm	**b**	32.3°				
12	**a**	18.1 cm^2	**b**	18.7 cm^2	**c**	693 mm^2		
13	5.10 cm							
14	**a**	6.29 cm	**b**	84.3°	**c**	5.73 cm	**d**	58.8°
15	15.3 cm							

Rearranging equations

A LEVEL LINKS
Scheme of work: 6a. Definition, differentiating polynomials, second derivatives
Textbook: Pure Year 1, 12.1 Gradients of curves

Key points

- To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
- You may need to factorise the terms containing the new subject.

Examples

Example 1 Make t the subject of the formula $v = u + at$.

$v = u + at$ $v - u = at$ $t = \frac{v-u}{a}$	**1** Get the terms containing t on one side and everything else on the other side. **2** Divide throughout by a.

Example 2 Make t the subject of the formula $r = 2t - \pi t$.

$r = 2t - \pi t$ $r = t(2 - \pi)$ $t = \frac{r}{2-\pi}$	**1** All the terms containing t are already on one side and everything else is on the other side. **2** Factorise as t is a common factor. **3** Divide throughout by $2 - \pi$.

Example 3 Make t the subject of the formula $\frac{t+r}{5} = \frac{3t}{2}$.

$\frac{t+r}{5} = \frac{3t}{2}$ $2t + 2r = 15t$ $2r = 13t$ $t = \frac{2r}{13}$	**1** Remove the fractions first by multiplying throughout by 10. **2** Get the terms containing t on one side and everything else on the other side and simplify. **3** Divide throughout by 13.

Example 4 Make t the subject of the formula $r = \dfrac{3t+5}{t-1}$.

$r = \dfrac{3t+5}{t-1}$	**1** Remove the fraction first by multiplying throughout by $t - 1$.
$r(t - 1) = 3t + 5$	**2** Expand the brackets.
$rt - r = 3t + 5$	
$rt - 3t = 5 + r$	**3** Get the terms containing t on one side and everything else on the other side.
$t(r - 3) = 5 + r$	**4** Factorise the LHS as t is a common factor.
$t = \dfrac{5+r}{r-3}$	**5** Divide throughout by $r - 3$.

Practice

Change the subject of each formula to the letter given in the brackets.

1 $C = \pi d$ [d]

2 $P = 2l + 2w$ [w]

3 $D = \dfrac{S}{T}$ [T]

4 $p = \dfrac{q-r}{t}$ [t]

5 $u = at - \dfrac{1}{2}t$ [t]

6 $V = ax + 4x$ [x]

7 $\dfrac{y-7x}{2} = \dfrac{7-2y}{3}$ [y]

8 $x = \dfrac{2a-1}{3-a}$ [a]

9 $x = \dfrac{b-c}{d}$ [d]

10 $h = \dfrac{7g-9}{2+g}$ [g]

11 $e(9 + x) = 2e + 1$ [e]

12 $y = \dfrac{2x+3}{4-x}$ [x]

13 Make r the subject of the following formulae.

a $A = \pi r^2$ **b** $V = \dfrac{4}{3}\pi r^3$ **c** $P = \pi r + 2r$ **d** $V = \dfrac{2}{3}\pi r^2 h$

14 Make x the subject of the following formulae.

a $\dfrac{xy}{z} = \dfrac{ab}{cd}$ **b** $\dfrac{4\pi cx}{d} = \dfrac{3z}{py^2}$

15 Make $\sin B$ the subject of the formula $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

16 Make $\cos B$ the subject of the formula $b^2 = a^2 + c^2 - 2ac \cos B$.

Extend

17 Make x the subject of the following equations.

a $\dfrac{p}{q}(sx+t) = x-1$ **b** $\dfrac{p}{q}(ax+2y) = \dfrac{3p}{q^2}(x-y)$

Answers

1 $d = \dfrac{C}{\pi}$

2 $w = \dfrac{P - 2l}{2}$

3 $T = \dfrac{S}{D}$

4 $t = \dfrac{q - r}{p}$

5 $t = \dfrac{2u}{2a - 1}$

6 $x = \dfrac{V}{a + 4}$

7 $y = 2 + 3x$

8 $a = \dfrac{3x + 1}{x + 2}$

9 $d = \dfrac{b - c}{x}$

10 $g = \dfrac{2h + 9}{7 - h}$

11 $e = \dfrac{1}{x + 7}$

12 $x = \dfrac{4y - 3}{2 + y}$

13 **a** $r = \sqrt{\dfrac{A}{\pi}}$ **b** $r = \sqrt[3]{\dfrac{3V}{4\pi}}$

c $r = \dfrac{P}{\pi + 2}$ **d** $r = \sqrt{\dfrac{3V}{2\pi h}}$

14 **a** $x = \dfrac{abz}{cdy}$ **b** $x = \dfrac{3dz}{4\pi cpy^2}$

15 $\sin B = \dfrac{b \sin A}{a}$

16 $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

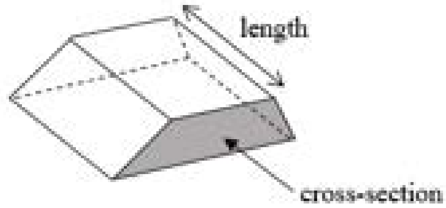
17 **a** $x = \dfrac{q + pt}{q - ps}$ **b** $x = \dfrac{3py + 2pqy}{3p - apq} = \dfrac{y(3 + 2q)}{3 - aq}$

edexcel

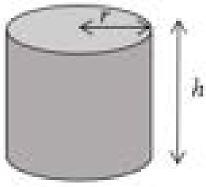
Volume and surface area of 3D shapes

A LEVEL LINKS
Scheme of work: 6b. Gradients, tangents, normals, maxima and minima

Key points

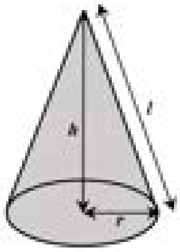


- Volume of a prism = cross-sectional area × length.
- The surface area of a 3D shape is the total area of all its faces.
- Volume of a pyramid = $\frac{1}{3}$ × area of base × vertical height.



- Volume of a cylinder = $\pi r^2 h$
- Total surface area of a cylinder = $2\pi r^2 + 2\pi rh$

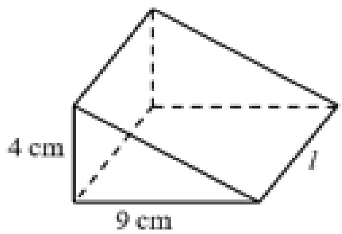
- Volume of a sphere = $\frac{4}{3}\pi r^3$
- Surface area of a sphere = $4\pi r^2$



- Volume of a cone = $\frac{1}{3}\pi r^2 h$
- Total surface area of a cone = $\pi rl + \pi r^2$

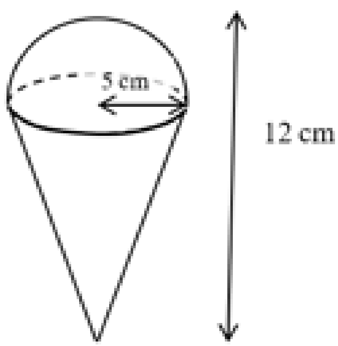
Examples

Example 1 The triangular prism has volume 504 cm^3. Work out its length.



$V = \frac{1}{2}bhl$	**1** Write out the formula for the volume of a triangular prism.
$504 = \frac{1}{2} \times 9 \times 4 \times l$	**2** Substitute known values into the formula.
$504 = 18 \times l$	**3** Simplify
$l = 504 \div 18$	**4**
$= 28$ cm	**5**

Example 2 Calculate the volume of the 3D solid. Give your answer in terms of π.

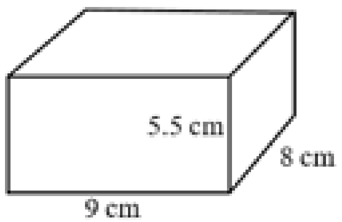


Total volume = volume of hemisphere + Volume of cone $= \frac{1}{2}$ of $\frac{4}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$	**1** The solid is made up of a hemisphere radius 5 cm and a cone with radius 5 cm and height $12 - 5 = 7$ cm.
Total volume $= \frac{1}{2} \times \frac{4}{3} \times \pi \times 5^3 + \frac{1}{3} \times \pi \times 5^2 \times 7$	**2** Substitute the measurements into the formula for the total volume.
$= \frac{425}{3}\pi \text{ cm}^3$	**3** Remember the units.

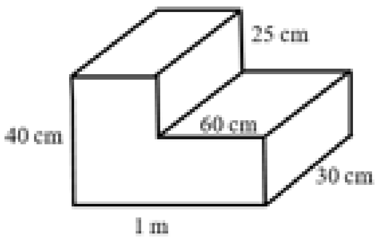
Practice

1 Work out the volume of each solid.
Leave your answers in terms of π where appropriate.

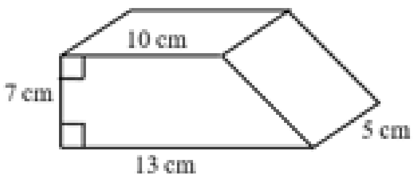
a



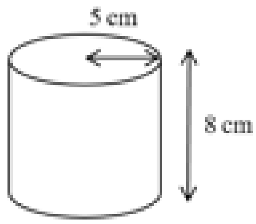
b



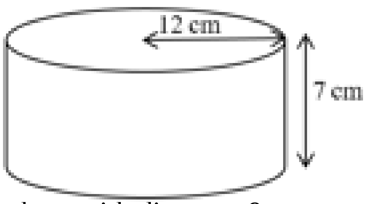
c



d



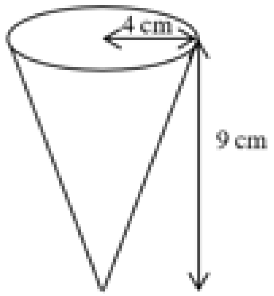
e

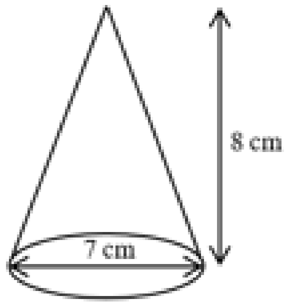


f a sphere with radius 7 cm

g a sphere with diameter 9 cm

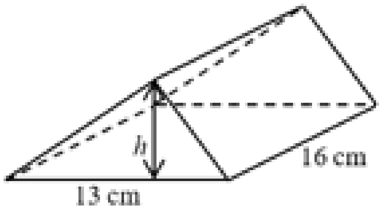
h a hemisphere with radius 3 cm

i 

j 

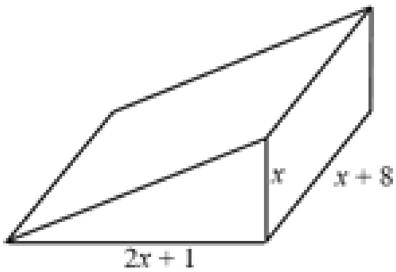
2 A cuboid has width 9.5 cm, height 8 cm and volume 1292 cm^3.
Work out its length.

3 The triangular prism has volume 1768 cm^3.
Work out its height.

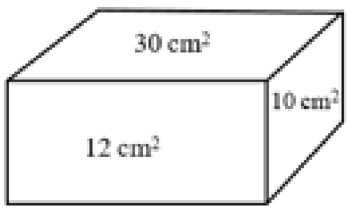


Extend

4 The diagram shows a solid triangular prism.
All the measurements are in centimetres.
The volume of the prism is V cm^3.
Find a formula for V in terms of x.
Give your answer in simplified form.



5 The diagram shows the area of each of three faces of a cuboid.
The length of each edge of the cuboid is a whole number of centimetres.
Work out the volume of the cuboid.



6 The diagram shows a large catering size tin of beans in the shape of a cylinder.

The tin has a radius of 8 cm and a height of 15 cm.

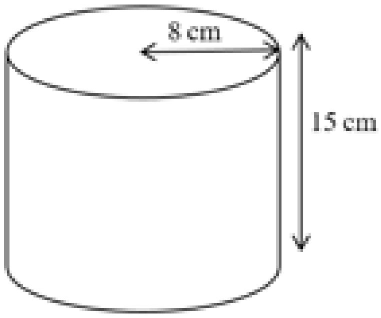
A company wants to make a new size of tin.

The new tin will have a radius of 6.7 cm.

It will have the same volume as the large tin.

Calculate the height of the new tin.

Give your answer correct to one decimal place.



7 The diagram shows a sphere and a solid cylinder.

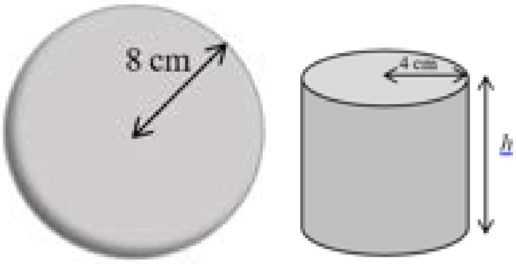
The sphere has radius 8 cm.

The solid cylinder has a base radius of 4 cm and a height of h cm.

The total surface area of the cylinder is half the total surface area of the sphere.

Work out the ratio of the volume of the sphere to the volume of the cylinder.

Give your answer in its simplest form.

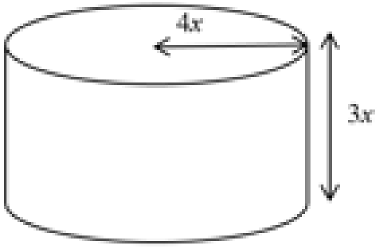


8 The diagram shows a solid metal cylinder.

The cylinder has base radius $4x$ and height $3x$.

The cylinder is melted down and made into a sphere of radius r.

Find an expression for r in terms of x.



Answers

1 **a** $V = 396 \text{ cm}^3$ **b** $V = 75\,000 \text{ cm}^3$

c $V = 402.5 \text{ cm}^3$ **d** $V = 200\pi \text{ cm}^3$

e $V = 1008\pi \text{ cm}^3$ **f** $V = \frac{1372}{3}\pi \text{ cm}^3$

g $V = 121.5\pi \text{ cm}^3$ **h** $V = 18\pi \text{ cm}^3$

i $V = 48\pi \text{ cm}^3$ **j** $V = \frac{98}{3}\pi \text{ cm}^3$

2 17 cm

3 17 cm

4 $V = x^3 + \frac{17}{2}x^2 + 4x$

5 60 cm^3

6 21.4 cm

7 32 : 9

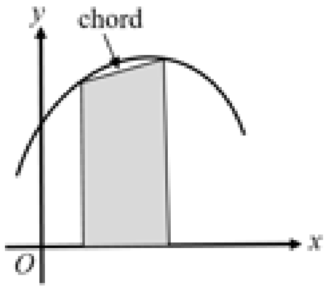
8 $r = \sqrt[3]{36x}$

Area under a graph

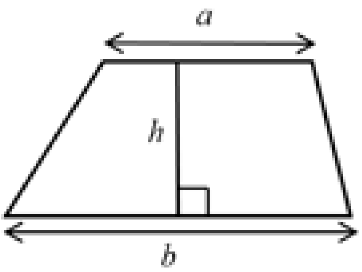
A LEVEL LINKS
Scheme of work: 7b. Definite integrals and areas under curves

Key points

- To estimate the area under a curve, draw a chord between the two points you are finding the area between and straight lines down to the horizontal axis to create a trapezium.
  The area of the trapezium is an approximation for the area under a curve.

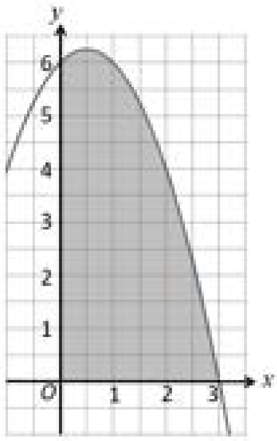


- The area of a trapezium = $\frac{1}{2}h(a+b)$



Examples

Example 1 Estimate the area of the region between the curve $y = (3 - x)(2 + x)$ and the x-axis from $x = 0$ to $x = 3$.
Use three strips of width 1 unit.



x	0	1	2	3
***y* = (3 − *x*)(2 + *x*)**	6	6	4	0

Trapezium 1:
$a_1 = 6 - 0 = 6$, $b_1 = 6 - 0 = 6$
Trapezium 2:
$a_2 = 6 - 0 = 6$, $b_2 = 4 - 0 = 4$
Trapezium 3:
$a_3 = 4 - 0 = 4$, $a_3 = 0 - 0 = 0$

1 Use a table to record the value of y on the curve for each value of x.

2 Work out the dimensions of each trapezium. The distances between the y-values on the curve and the x-axis give the values for a.

(continued on next page)

$\frac{1}{2}h(a_1 + b_1) = \frac{1}{2} \times 1(6+6) = 6$

$\frac{1}{2}h(a_2 + b_2) = \frac{1}{2} \times 1(6+4) = 5$

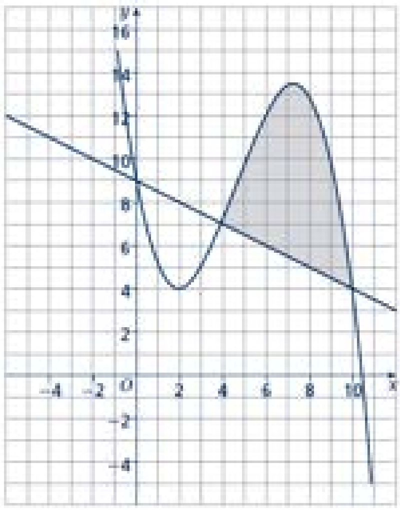
$\frac{1}{2}h(a_3 + b_3) = \frac{1}{2} \times 1(4+0) = 2$

3 Work out the area of each trapezium. h = 1 since the width of each trapezium is 1 unit.

Area = 6 + 5 + 2 = 13 units^2

4 Work out the total area. Remember to give units with your answer.

Example 2 Estimate the shaded area.
Use three strips of width 2 units.



x	4	6	8	10
y	7	12	13	4

1 Use a table to record y on the curve for each value of x.

x	4	6	8	10
y	7	6	5	4

2 Use a table to record y on the straight line for each value of x.

Trapezium 1:
$a_1 = 7 - 7 = 0$, $b_1 = 12 - 6 = 6$
Trapezium 2:
$a_2 = 12 - 6 = 6$, $b_2 = 13 - 5 = 8$
Trapezium 3:
$a_3 = 13 - 5 = 8$, $a_3 = 4 - 4 = 0$

3 Work out the dimensions of each trapezium. The distances between the y-values on the curve and the y-values on the straight line give the values for a.

$\frac{1}{2}h(a_1 + b_1) = \frac{1}{2} \times 2(0+6) = 6$

$\frac{1}{2}h(a_2 + b_2) = \frac{1}{2} \times 2(6+8) = 14$

$\frac{1}{2}h(a_3 + b_3) = \frac{1}{2} \times 2(8+0) = 8$

4 Work out the area of each trapezium. h = 2 since the width of each trapezium is 2 units.

Area = 6 + 14 + 8 = 28 units^2

5 Work out the total area. Remember to give units with your answer.

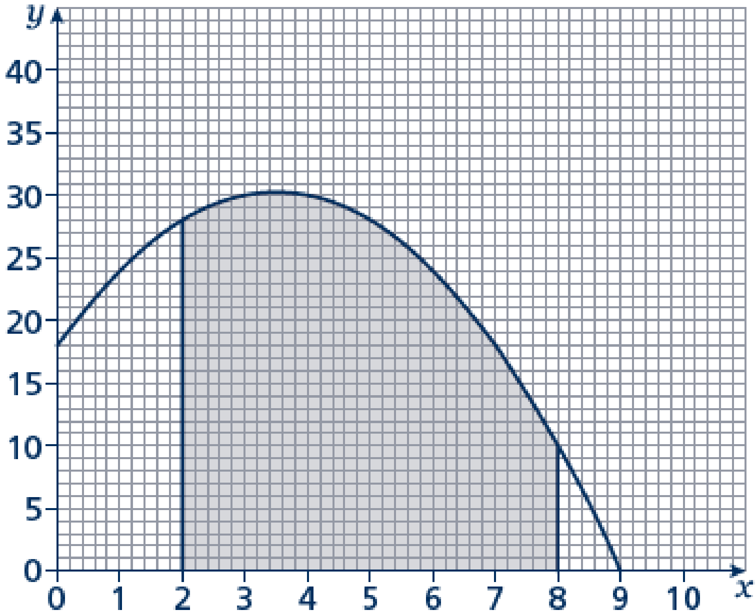
Practice

Hint:

For a full answer, remember to include 'units^2'.

1 Estimate the area of the region between the curve $y = (5 - x)(x + 2)$ and the x-axis from $x = 1$ to $x = 5$.
Use four strips of width 1 unit.

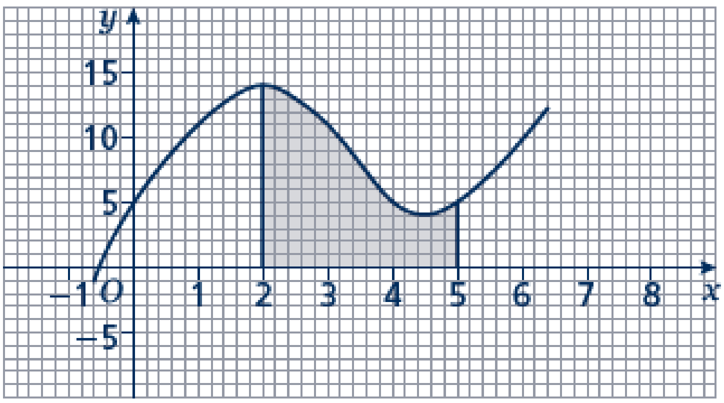
2 Estimate the shaded area shown on the axes.
Use six strips of width 1 unit.



3 Estimate the area of the region between the curve $y = x^2 - 8x + 18$ and the x-axis from $x = 2$ to $x = 6$.
Use four strips of width 1 unit.

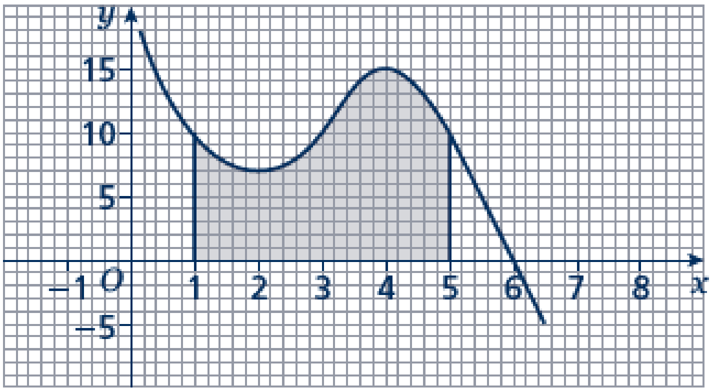
4 Estimate the shaded area.

Use six strips of width $\frac{1}{2}$ unit.



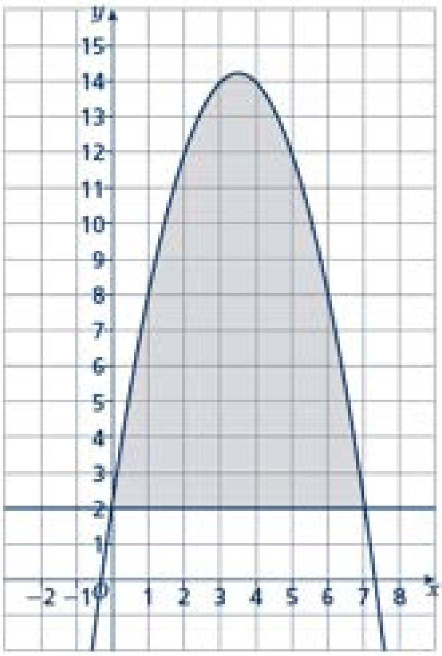
5 Estimate the area of the region between the curve $y = -x^2 - 4x + 5$ and the x-axis from $x = -5$ to $x = 1$.
Use six strips of width 1 unit.

6 Estimate the shaded area.
Use four strips of equal width.



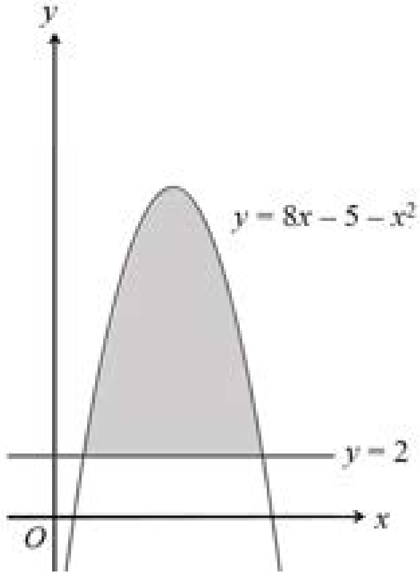
7 Estimate the area of the region between the curve $y = -x^2 + 2x + 15$ and the x-axis from $x = 2$ to $x = 5$.
Use six strips of equal width.

8 Estimate the shaded area.
Use seven strips of equal width.

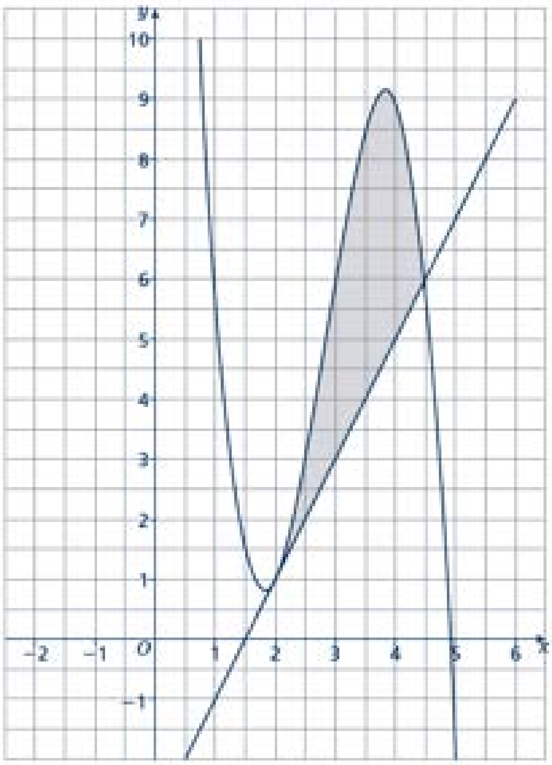


Extend

9 The curve $y = 8x - 5 - x^2$ and the line $y = 2$ are shown in the sketch.
Estimate the shaded area using six strips of equal width.



10 Estimate the shaded area using five strips of equal width.



Answers

1 34 units^2

2 149 units^2

3 14 units^2

4 $25\frac{1}{4}$ units^2

5 35 units^2

6 42 units^2

7 $26\frac{7}{8}$ units^2

8 56 units^2

9 35 units^2

10 $6\frac{1}{4}$ units^2

Useful Videos to Support 6th Form Induction for Mathematics

If you are not sure about a topic then please use the Corbett Maths video listed below. You will need to go to "corbettmaths.com". It is a free website and you do not need a log in. Remember that you still have access to Sparx Maths and the useful videos there that can help support you.

Examsolutions.net, Mathsgenie.co.uk and TLMaths are also useful websites that have videos to support GCSE Maths as well A level Maths.

- Rational and irrational numbers CM 230
- Rules of fractions (+,-,x,÷) CM 133 – 146 (various skills)
- Negative indices CM 175
- Fractional indices CM 173
- Laws of indices CM 174
- Factorising single brackets CM 117
- Factorising double brackets CM 118, 119, 119a
- Algebraic fractions CM 21, 22, 23, 24
- Rearranging formulae CM 7, 8
- Solving quadratic equations CM 266
- Solving quadratic equations using surds CM 267, 267a, 267b
- Difference of two squares CM 120
- Further work with surds CM 305, 306, 307, 308
- Solving linear simultaneous equations CM 295, 296
- Linear and quadratic simultaneous eqn CM 298
- Gradient of a straight line CM 189
- Y = mx + c CM 187, 191, 194, 195
- Parallel and perpendicular lines CM 196, 197
- Distance between two points CM 88
- Midpoint of two points on a line CM 87, 198
- Trigonometry (SOHCAHTOA) CM 329, 330, 331
- Sine rule CM 333, 334, 334a
- Cosine rule CM 335, 336
- Circle theorems CM 64, 65, 65a, 65b, 65c, 65d, 65e, 65f
- Cones CM 314, 359
- Spheres CM 313, 361
- Direct and inverse proportion CM 254, 255
- Equation of a circle CM 12
- Solving linear inequalities CM 178, 179
- Solving quadratic inequalities CM 378

Other videos that will be helpful before you start the A level Mathematics course:

- Expanding three brackets CM 15
- Roots and turning points of a quadratic CM 265, 265a, 265b
- Completing the square CM 267a, 371
- Area of a triangle using Sine CM 337
- Trigonometry in 3D CM 332
- Trigonometric graphs CM 338, 339, 340
- Transformation of functions CM 323, 324

For Students wishing to explore Mathematics beyond the A level course we have some suggestions:

Further reading suggestions:

- How to Study for a Maths Degree (Lara Alcock, OUP)
- Makers of Mathematics (S Hollingdale, Penguin)
- Alan Turing, the Enigma (A Hodges, Vintage)
- The Man Who Knew Infinity (R Kanigel, Abacus)
- A Mathematician's Apology (G H Hardy, CUP)
- Surely You're Joking, Mr Feynmann (R P Feynmann, Arrow)
- Fermat's Last Theorem (Simon Singh, Fourth Estate)
- The Music of the Primes (Marcus du Sautoy, Harper-Collins)
- Incredible Numbers (Ian Stewart, Profile Books)
- The Colossal Book of Mathematics (Martin Gardner, Norton)
- Game, Set and Math (Ian Stewart, Penguin)
- How to think like a mathematician (Kevin Houston, CUP)
- What is Mathematics? (Courant& Robbins, OUP)
- The Pleasures of Counting (T W Korner, CUP)
- Chaos (Gleick, Minerva/Random House)
- Alex's Adventures in Numberland (Alex Bellos, Bloomsbury)
- Things to Make and Do in the Fourth Dimension (Matt Parker, Penguin)
- 17 Equations That Changed the World (Ian Stewart, Profile Books)

Blogs and Twitter handles to follow:

- @Gapminder
- @bletchleypark
- @DataSciFact
- @ONS
- @signmagazine
- @RoyalStatSoc
- @ProbFact
- @AlgebraFact
- @MathsJam
- @mathshistory
- @nrichmaths
- @OxUniMaths
- @CambridgeMaths
- @UKMathsTrust
- @Mathematical_A
- @evelynjlamb@
- @stevenstrogatz
- @1to9puzzle
- @drewfoster0
- @Mazed70

Useful websites:

- http://ww1.furthermaths.org/?subid1=992d54b6-014b-11ea-af89-cb59abf8057a
- https://www.drfrostmaths.com/sow.php?year=A%20Level%202017&term=Main
- https://amsp.org.uk/students/a-level
- https://amsp.org.uk/students/a-level-further
- http://www.cut-the-knot.org/
- https://www.khanacademy.org/
- https://nrich.maths.org/
- https://plus.maths.org/content/
- https://brilliant.org/
- https://www.ukmt.org.uk/
- https://mathschallenge.net/
- http://www.qbyte.org/puzzles/
- https://a-levelmaths.com/
- http://mathcentre.ac.uk/
- https://www.vivaxsolutions.com/

Films:

- Stand and Deliver
- The Man Who Knew Infinity
- A Beautiful Mind
- Pi
- Good Will Hunting
- The Imitation Game
- A Brief History of Time
- X + Y
- Codebreaker
- N Is a Number: A Portrait of Paul Erdös
- Fermat's Room
- The Oxford Murders
- Hidden Figures

Online Lectures:

- https://www.maths.ox.ac.uk/study-here/prospective-undergraduates-old/outreach/online-lectures
- https://www.maths.ox.ac.uk/events/public-lectures-events#PublicLecturesOnline
- https://www.birmingham.ac.uk/schools/mathematics/news-and-events/birmingham-popular-maths-lecture.aspx
- https://www.ted.com/topics/math
- https://www.gapminder.org/videos/the-joy-of-stats/
- https://www.admin.cam.ac.uk/whatson/index.shtml?utm_campaign=newsletters&utm_medium=email&utm_source=529573_Open%20Cambridge%20starts%20tomorrow%21&dm_i=6DCF,BCMD,2MY7EJ,1D742,1
  https://www.channeltalent.co.uk/event/mathematics-exploring-the-nature-applications-of-complex-numbers-university-tutorial/

GCSE PE → AQA A-Level PE Bridging Task

Summer Transition Project

Purpose:
To support your transition from GCSE PE to AQA A-Level PE by introducing key theoretical concepts in each section of the course. Each task is designed to develop your knowledge, research, and analytical skills.

Instructions:
Complete **all three tasks** below (one for each A-Level PE section). Bring your completed work to your first A-Level PE lesson in September. Work can be typed or handwritten neatly.

Task 1 – Applied Anatomy and Physiology ***(Section A)***

Topic: The Cardiovascular System and the Effects of Physical Activity

Objective: Understand how the heart functions and responds to physical activity.

Your Task:
Create a two-part summary report or a two-page spread that includes:

Part A – Structure and Function

- Labelled diagram of the heart
- Blood flow through the heart and lungs (double circulatory system)
- The cardiac conduction system (SA node, AV node, bundle of His, Purkinje fibres)

Part B – Impact of Physical Activity

- Definitions: **Heart rate (HR)**, **stroke volume (SV)**, **cardiac output (Q = HR × SV)**
- The **short-term** effects of exercise on these three variables
- The **long-term** adaptations to regular aerobic training (e.g., lower resting HR, increased SV, cardiac hypertrophy)
- Include a graph showing HR response to exercise (rest, exercise, recovery)

Stretch Task:
Use sport-specific examples (e.g., running, swimming, football) to explain how the cardiovascular system adapts over time.

Task 2 – Skill Acquisition and Sports Psychology ***(Section B)***

Topic: Skill Classification and Practice Methods

Objective: Understand how skills are categorised and how athletes learn and improve them.

Your Task:
Create a visual presentation (e.g., poster, slides, or infographic) that covers:

- The **skill classification continua**:
  - Open vs. closed
  - Gross vs. fine
  - Simple vs. complex
- Definitions and sporting examples for each continuum
- Types of practice: **massed**, **distributed**, **fixed**, **variable**
- Types of feedback: **intrinsic**, **extrinsic**, **concurrent**, **terminal**

Apply It:
Choose **two skills** from a sport you play. For each:

- Classify the skill using the continua
- Recommend the best type of practice and feedback for improving the skill

Task 3 – Sport and Society ***(Section C)***

Topic: Women's Participation in Sport – Football, Olympics & Tennis

Objective: Explore how gender equality in sport has progressed and where barriers still exist.

Your Task:
Write an essay (400–600 words) answering the question:
"How has women's participation in sport developed over time, and what barriers still exist today?"

You must include:

- **Women's Football**: e.g., growth of the Women's Super League (WSL), England's Euro 2022 win, FIFA Women's World Cup
- **Women in the Olympics**: e.g., first female participants, inclusion of more events, media coverage gaps
- **Women's Tennis**: e.g., Billie Jean King and the "Battle of the Sexes", equal pay at Grand Slams, Serena Williams' impact

Also include:

- Historical restrictions on female participation
- Key social changes and campaigns promoting equality
- Current challenges (e.g., media representation, funding, stereotypes)

Extension:
Why is it important to study sport in its historical and societal context at A-Level?

Transition Pack for A Level Physics

Get ready for A-level!

A guide to help you get ready for A-level Physics, including everything from topic guides to days out and online learning courses.

Commissioned by The PiXL Club Ltd. February 2016

Please note: these resources are non-board specific. Please direct your students to the specifics of where this knowledge and skills most apply.

So you are considering A Level Physics?

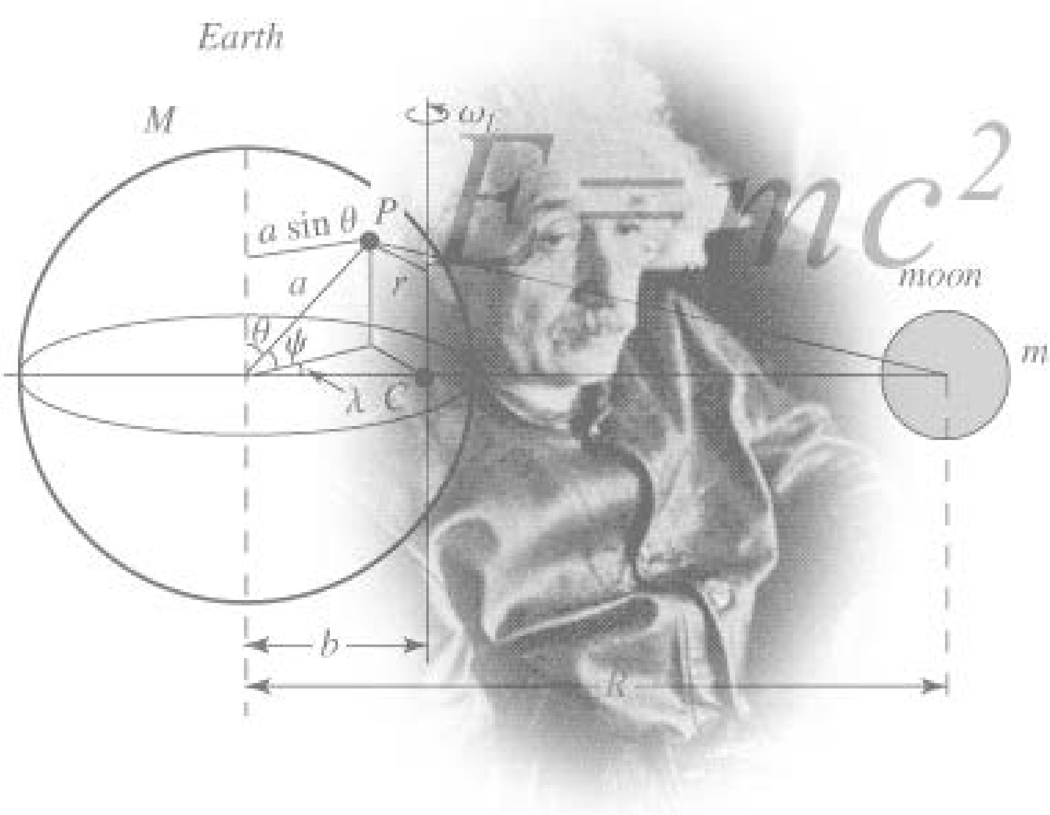


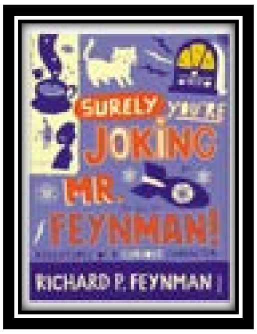
Figure 1 http://scienceworld.wolfram.com/physics/images/main-physics.gif

This pack contains a programme of activities and resources to prepare you to start an A level in Physics in September. It is aimed to be used after you complete your GCSE, throughout the remainder of the Summer term and over the Summer Holidays to ensure you are ready to start your course in September.

Book Recommendations

Below is a selection of books that should appeal to a physicist – someone with an enquiring mind who wants to understand the universe around us. None of the selections are textbooks full of equation work (there will be plenty of time for that!) instead each provides insight to either an application of physics or a new area of study that you will be meeting at A Level for the first time.

1. **Surely You're Joking Mr Feynman: Adventures of a Curious Character**

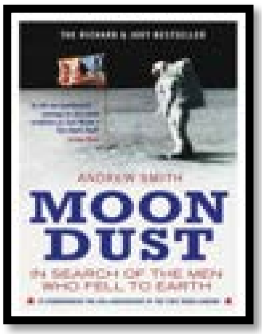


ISBN - 009917331X - Richard Feynman was a Nobel Prize winning Physicist. In my opinion he epitomises what a Physicist is. By reading this books you will get insight into his life's work including the creation of the first atomic bomb and his bongo playing adventures and his work in the field of particle physics.

(Also available on Audio book).

https://www.waterstones.com/books/search/term/surely+youre+joking+mr+feynman++adventures+of+a+curious+character

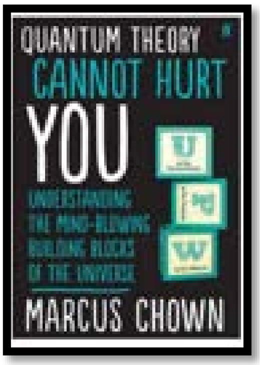
2. **Moondust: In Search of the Men Who Fell to Earth**



ISBN – 1408802384 - One of the greatest scientific achievements of all time was putting mankind on the surface of the moon. Only 12 men made the trip to the surface, at the time of writing the book only 9 are still with us. The book does an excellent job of using the personal accounts of the 9 remaining astronauts and many others involved in the space program at looking at the whole space-race era, with hopefully a new era of space flight about to begin as we push on to put mankind on Mars in the next couple of decades.

https://www.waterstones.com/books/search/term/moondust++in+search+of+the+men+who+fell+to+earth

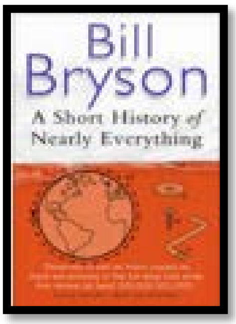
3. **Quantum Theory Cannot Hurt You: Understanding the Mind-Blowing Building Blocks of the Universe**



ISBN - 057131502X - Any Physics book by Marcus Chown is an excellent insight into some of the more exotic areas of Physics that require no prior knowledge. In your first year of A-Level study you will meet the quantum world for the first time. This book will fill you with interesting facts and handy analogies to whip out to impress your peers!

https://www.waterstones.com/book/quantum-theory-cannot-hurt-you/marcus-chown/9780571315024

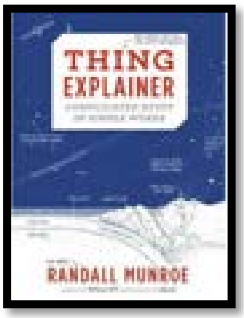
4. **A Short History of Nearly Everything**



ISBN – 0552997048 - A modern classic. Popular science writing at its best. A Short History of Nearly Everything Bill Bryson's quest to find out everything that has happened from the Big Bang to the rise of civilization - how we got from there, being nothing at all, to here, being us. Hopefully by reading it you will gain an awe-inspiring feeling of how everything in the universe is connected by some fundamental laws.

https://www.waterstones.com/books/search/term/a+short+history+of+nearly+everything

5. **Thing Explainer: Complicated Stuff in Simple Words**



ISBN – 1408802384 - This final recommendation is a bit of a wild-card – a book of illustrated cartoon diagrams that should appeal to the scientific side of everyone. Written by the creator of online comic XTCD (a great source of science humour) is a book of blueprints from everyday objects such as a biro to the Saturn V rocket and an atom bomb, each one meticulously explained BUT only with the most common 1000 words in the English Language. This would be an excellent coffee table book in the home of every scientist.

https://www.waterstones.com/book/thing-explainer/randall-munroe/9781473620919

Movie / Video Clip Recommendations

Hopefully you'll get the opportunity to soak up some of the Sun's rays over the summer – synthesising some important Vitamin-D – but if you do get a few rainy days where you're stuck indoors here are some ideas for films to watch or clips to find online.

Science Fictions Films

1. **Moon (2009)**
2. **Gravity (2013)**
3. **Interstellar (2014)**
4. **The Imitation Game (2015)**
5. **The Prestige (2006)**

Online Clips / Series

1. **Minute Physics –** Variety of Physics questions explained simply (in felt tip) in a couple of minutes. Addictive viewing that will have you watching clip after clip – a particular favourite of mine is "Why is the Sky Dark at Night?"

   https://www.youtube.com/user/minutephysics

2. **Wonders of the Universe / Wonders of the Solar System –** Both available of Netflix as of 17/4/16 – Brian Cox explains the Cosmos using some excellent analogies and wonderful imagery.

3. **Shock and Awe, The Story of Electricity –** A 3 part BBC documentary that is essential viewing if you want to see how our lives have been transformed by the ideas of a few great scientists a little over 100 years ago. The link below takes you to a stream of all three parts joined together but it is best watched in hourly instalments. Don't forget to boo when you see Edison. (alternatively watch any Horizon documentary – loads of choice on Netflix and the I-Player)

   https://www.youtube.com/watch?v=Gtp51eZkwoI

4. **NASA TV –** Online coverage of launches, missions, testing and the ISS. Plenty of clips and links to explore to find out more about applications of Physics in Space technology.

   http://www.nasa.gov/multimedia/nasatv/

5. **The Fantastic Mr. Feynman –** I recommended the book earlier, I also cannot recommend this 1 hour documentary highly enough. See the life's work of the "great explainer", a fantastic mind that created mischief in all areas of modern Physics.

   https://www.youtube.com/watch?v=LyqleIxXTpw

Research activity

To get the best grades in A Level Physics you will have to get good at completing independent research and making your own notes on difficult topics. Below are links to 5 websites that cover some interesting Physics topics.

Using the Cornell notes system: http://coe.jmu.edu/learningtoolbox/cornellnotes.html make 1 page of notes **from each site** covering a topic of your choice.

a) http://home.cern/about

   CERN encompasses the Large Hadron Collider (LHC) and is the largest collaborative science experiment ever undertaken. Find out about it here and make a page of suitable notes on the accelerator.

b) http://joshworth.com/dev/pixelspace/pixelspace_solarsystem.html

   The solar system is massive and its scale is hard to comprehend. Have a look at this award winning website and make a page of suitable notes.

c) https://phet.colorado.edu/en/simulations/category/html

   PhET create online Physics simulations when you can complete some simple experiments online. Open up the resistance of a wire html5 simulation. Conduct a simple experiment and make a one page summary of the experiment and your findings.

d) http://climate.nasa.gov/

NASA's Jet Propulsion Laboratory has lots of information on Climate Change and Engineering Solutions to combat it. Have a look and make notes on an article of your choice.

e) http://www.livescience.com/46558-laws-of-motion.html

Newton's Laws of Motion are fundamental laws for the motion of all the object we can see around us. Use this website and the suggested further reading links on the webpage to make your own 1 page of notes on the topics.

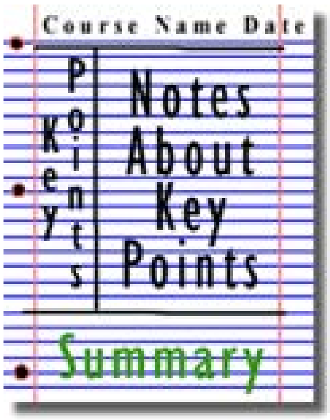


Figure 2: http://coe.jmu.edu/learningtoolbox/images/noteb4.gif

Pre-Knowledge Topics

Below are ten topics that are essential foundations for you study of A-Level Physics. Each topics has example questions and links where you can find our more information as you prepare for next year.

Symbols and Prefixes

Prefix	Symbol	Power of ten
Nano	n	x 10^{-9}
Micro	µ	x 10^{-6}
Milli	m	x 10^{-3}
Centi	c	x 10^{-2}
Kilo	k	x 10^{3}
Mega	M	x 10^{6}
Giga	G	x 10^{9}

At A level, unlike GCSE, you need to remember all symbols, units and prefixes. Below is a list of quantities you may have already come across and will be using during your A level course

Quantity	Symbol	Unit
Velocity	v	ms^{-1}
Acceleration	a	ms^{-2}
Time	t	S
Force	F	N
Resistance	R	Ω
Potential difference	V	V
Current	I	A
Energy	E or W	J
Pressure	P	Pa
Momentum	p	$kgms^{-1}$
Power	P	W
Density	ρ	kgm^{-3}
Charge	Q	C

Solve the following:

1. How many metres in 2.4 km?
2. How many joules in 8.1 MJ?
3. Convert 326 GW into W.
4. Convert 54 600 mm into m.
5. How many grams in 240 kg?
6. Convert 0.18 nm into m.
7. Convert 632 nm into m. Express in standard form.
8. Convert 1002 mV into V. Express in standard form.
9. How many eV in 0.511 MeV? Express in standard form.
10. How many m in 11 km? Express in standard form.

Standard Form

At A level quantity will be written in standard form, and it is expected that your answers will be too.

This means answers should be written as ….x 10^y. E.g. for an answer of 1200kg we would write 1.2 x 10^3kg. For more information visit: www.bbc.co.uk/education/guides/zc2hsbk/revision

1. Write 2530 in standard form.
2. Write 280 in standard form.
3. Write 0.77 in standard form.
4. Write 0.0091 in standard form.
5. Write 1 872 000 in standard form.
6. Write 12.2 in standard form.
7. Write 2.4 x 10^2 as a normal number.
8. Write 3.505 x 10^1 as a normal number.
9. Write 8.31 x 10^6 as a normal number.
10. Write 6.002 x 10^2 as a normal number.
11. Write 1.5 x 10^{-4} as a normal number.
12. Write 4.3 x 10^3 as a normal number.

Rearranging formulae

This is something you will have done at GCSE and it is crucial you master it for success at A level. For a recap of GCSE watch the following links:

www.khanacademy.org/math/algebra/one-variable-linear-equations/old-school-equations/v/solving-for-a-variable

www.youtube.com/watch?v=_WWgc3ABSj4

Rearrange the following:

1. E=m x g x h to find h
2. Q= I x t to find I
3. E = ½ m v^2 to find m
4. E = ½ m v^2 to find v
5. v = u + at to find u
6. v = u + at to find a
7. $v^2 = u^2$ +2as to find s
8. $v^2 = u^2$ +2as to find u

Significant figures

At A level you will be expected to use an appropriate number of significant figures in your answers. The number of significant figures you should use is the same as the number of significant figures in the data you are given. You can never be more precise than the data you are given so if that is given to 3 significant your answer should be too. E.g. Distance = 8.24m, time = 1.23s therefore speed = 6.75m/s

The website below summarises the rules and how to round correctly.

http://www.purplemath.com/modules/rounding2.htm

Give the following to 3 significant figures:

1. 3.4527
2. 40.691
3. 0.838991
4. 1.0247
5. 59.972

Calculate the following to a suitable number of significant figures:

6. 63.2/78.1
7. 39+78+120
8. (3.4+3.7+3.2)/3
9. 0.0256 x 0.129
10. 592.3/0.1772

Atomic Structure

You will study nuclear decay in more detail at A level covering the topics of radioactivity and particle physics. In order to explain what happens you need to have a good understanding of the model of the atom. You need to know what the atom is made up of, relative charges and masses and how sub atomic particles are arranged.

The following video explains how the current model was discovered
www.youtube.com/watch?v=wzALbzTdnc8

Describe the model used for the structure of an atom including details of the individual particles that make up an atom and the relative charges and masses of these particles. You may wish to include a diagram and explain how this model was discovered by Rutherford

Recording Data

Whilst carrying out a practical activity you need to write all your raw results into a table. Don't wait until the end, discard anomalies and then write it up in neat.

Tables should have column heading and units in this format quantity/unit e.g. length /mm

All results in a column should have the same precision and if you have repeated the experiment you should calculate a mean to the same precision as the data.

Below are link to practical handbooks so you can familiarise yourself with expectations.

http://filestore.aqa.org.uk/resources/physics/AQA-7407-7408-PHBK.PDF

http://www.ocr.org.uk/Images/295483-practical-skills-handbook.pdf

http://www.ocr.org.uk/Images/295483-practical-skills-handbook.pdf

Below is a table of results from an experiment where a ball was rolled down a ramp of different lengths. A ruler and stop clock were used.

1) Identify the errors the student has made.

	Time			
Length/cm	**Trial 1**	**Trial 2**	**Trial 3**	**Mean**
10	1.45	1.48	1.46	1.463
22	2.78	2.72	2.74	2.747
30	4.05	4.01	4.03	4.03
41	5.46	5.47	5.46	5.463
51	7.02	6.96	6.98	6.98
65	8.24	9.68	8.24	8.72
70	9.01	9.02	9.0	9.01

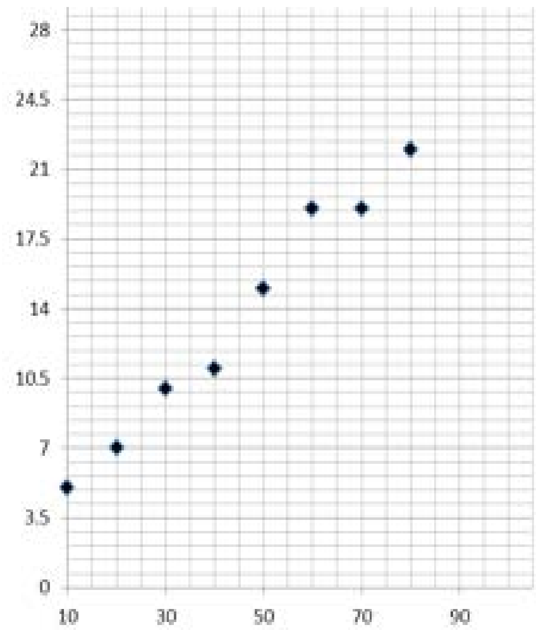
Graphs

After a practical activity the next step is to draw a graph that will be useful to you. Drawing a graph is a skill you should be familiar with already but you need to be extremely vigilant at A level. Before you draw your graph to need to identify a suitable scale to draw taking the following into consideration:

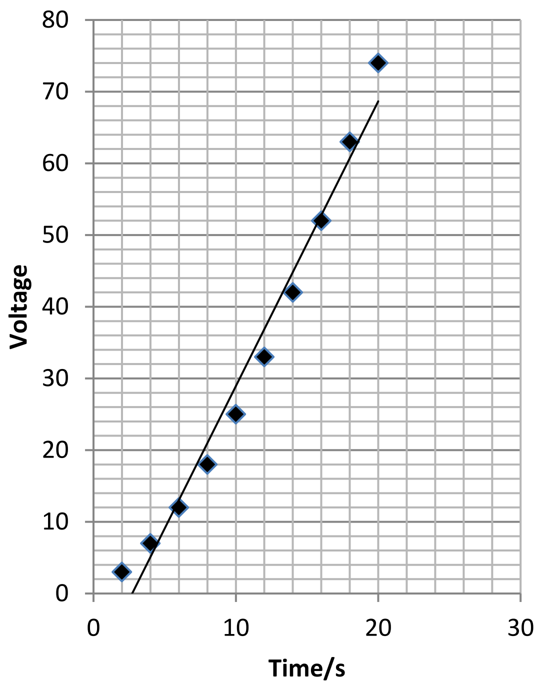
- the maximum and minimum values of each variable
- whether 0.0 should be included as a data point; graphs don't need to show the origin, a false origin can be used if your data doesn't start near zero.
- the plots should cover at least half of the grid supplied for the graph.
- the axes should use a sensible scale e.g. multiples of 1,2, 5 etc)

Identify how the following graphs could be improved

Graph 1



Graph 2



Forces and Motion

At GCSE you studied forces and motion and at A level you will explore this topic in more detail so it is essential you have a good understanding of the content covered at GCSE. You will be expected to describe, explain and carry calculations concerning the motion of objects. The websites below cover Newton's laws of motion and have links to these in action.

http://www.physicsclassroom.com/Physics-Tutorial/Newton-s-Laws

http://www.sciencechannel.com/games-and-interactives/newtons-laws-of-motion-interactive/

Sketch a velocity-time graph showing the journey of a skydiver after leaving the plane to reaching the ground.

Mark on terminal velocity.

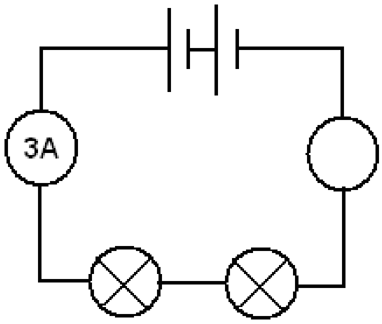
Electricity

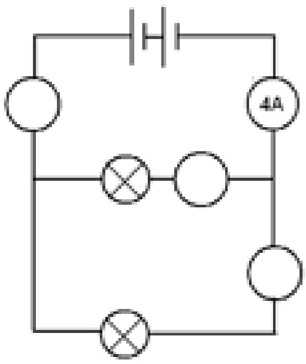
At A level you will learn more about how current and voltage behave in different circuits containing different components. You should be familiar with current and voltage rules in a series and parallel circuit as well as calculating the resistance of a device.

http://www.allaboutcircuits.com/textbook/direct-current/chpt-1/electric-circuits/

http://www.physicsclassroom.com/class/circuits

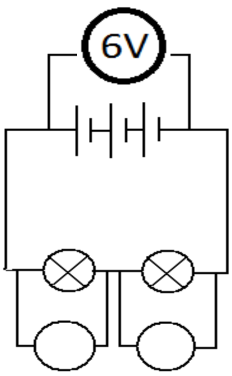
1a) Add the missing ammeter readings on the circuits below.

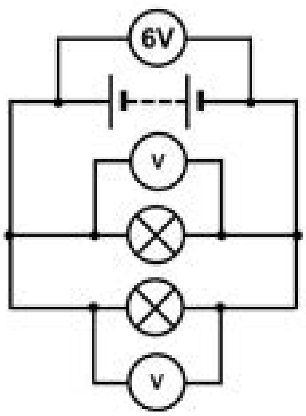




b) Explain why the second circuit has more current flowing than the first.

2) Add the missing potential differences to the following circuits





Waves

You have studied different types of waves and used the wave equation to calculate speed, frequency and wavelength. You will also have studied reflection and refraction.

Use the following links to review this topic.

http://www.bbc.co.uk/education/clips/zb7gkqt

https://www.khanacademy.org/science/physics/mechanical-waves-and-sound/mechanical-waves/v/introduction-to-waves

https://www.khanacademy.org/science/physics/mechanical-waves-and-sound/mechanical-waves/v/introduction-to-waves

1) Draw a diagram showing the refraction of a wave through a rectangular glass block. Explain why the ray of light takes this path.

2) Describe the difference between a longitudinal and transverse waves and give an example of each

3) Draw a wave and label the wavelength and amplitude

Pre-Knowledge Topics Answers:

Symbols and prefixes

1. 2400
2. 8 100 000
3. 326 000 000 000
4. 54.6
5. 240 000
6. 1.8×10^{-8}
7. 6.32×10^{-7}
8. 1.002
9. 5.11×10^{-5}
10. 1.1×10^{4}

Standard Form:

1. 2.53
2. 2.8
3. 7.7
4. 9.1
5. 1.872
6. 1.22
7. 2400
8. 35.05
9. 8 310 000
10. 600.2
11. 0.00015
12. 4300

Rearranging formulae

1. h= E/ (m x g)
2. I = Q/t
3. m = (2 x E)/v^2 or E/(0.5 x v^2)
4. v= √((2 x E)/m)
5. u = v – at
6. a = (v-u)/t
7. s = (v^2 – u^2) / 2a
8. u = √(v^2-2as)

Significant figures

1. 3.35
2. 40.7
3. 0.839
4. 1.02
5. 60.0
6. 0.809
7. 237
8. 3.4
9. 0.00330
10. 3343

Atomic Structure

contains protons, neutrons and electrons

Relative charge:

protons are positive (+1)

electrons are negative (-1)

neutrons are uncharged (0)

Relative mass:

proton 1

neutron 1

electron (about) 1/2000

protons and neutrons make up the nucleus

the nucleus is positively charged

electrons orbit the nucleus at a relatively large distance from the nucleus

most of the atom is empty space

nucleus occupies a very small fraction of the volume of the atom

most of the mass of the atom is contained in the nucleus

total number of protons in the nucleus equals the total number of electrons orbiting it in an atom

Recording data

Time should have a unit next to it

Length can be measured to the nearest mm so should be 10.0, 22.0 etc

Length 65 trial 2 is an anomaly and should have been excluded from the mean

All mean values should be to 2 decimal places

Mean of length 61 should be 6.99 (rounding error)

Graphs

Graph 1:

Axis need labels

Point should be x not dots

Line of best fit is needed

y axis is a difficult scale

x axis could have begun at zero so the y-intercept could be found

Graph 2:

y-axis needs a unit

curve of best fit needed not a straight line

Point should be x not dots

Forces and motion

Graph to show acceleration up to a constant speed (labelled terminal velocity). Rate of acceleration should be decreasing. Then a large decrease in velocity over a short period of time (parachute opens), then a decreasing rate of deceleration to a constant speed (labelled terminal velocity)

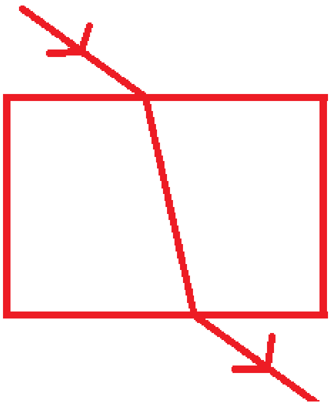
Electricity

1a) Series: 3A, Parallel top to bottom: 4A,2A,2A

b) Less resistance in the parallel circuit. Link to R=V/I. Less resistance means higher current.

2) Series: 3V, 3V, Parallel: 6V 6V

Waves

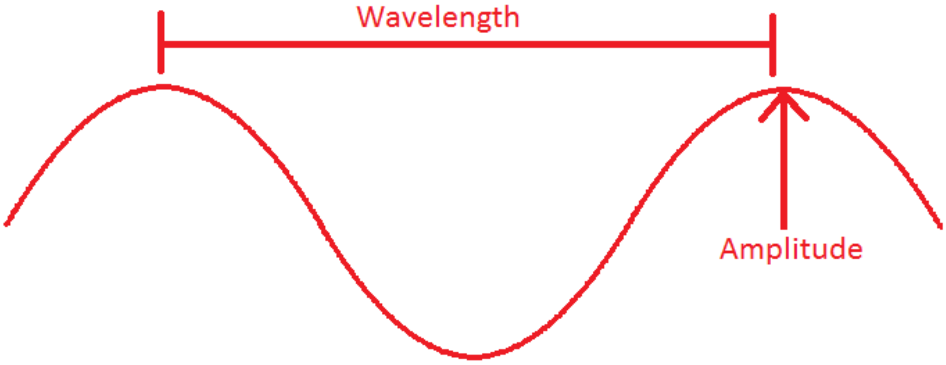


1) When light enters a more optically dense material it slows down and therefore bends towards the normal. The opposite happened when it leaves an optically dense material.

2) A longitudinal wave oscillates parallel to the direction of energy transfer (e.g. sound). A transverse waves oscillated perpendicular to the direction of energy transfer (e.g. light)

3)

.



Ideas for Day Trips

Here are some suggestions for some physics-themed days out for you to enjoy over the summer break. Try and have some fun as you prepare for two tough but rewarding years ahead!

Northern England and Scotland

1. **Jodrell Bank Observatory** – Cheshire – one of the largest moveable radio telescopes in the world and the location of the filming of the BBC's Stargazing Live. The site has both indoor and outdoor activities.
2. **MOSI** – Manchester – Massive free museum showing how science helped Britain lead the way through the industrial revolution. Contains hands on exhibits and displays and often host regular travelling exhibitions.
3. **Liverpool World Museum** / **Spaceport –** Liverpool/Wirral – Start the day off at an excellent family science museum with a top floor dedicated to astronomy including a planetarium. Take the ferry cross the Mersey to another family friendly museum dedicated to spaceflight.
4. **Kielder Observatory** – Northumberland – Book ahead at this popular observatory in the midst of the darkest night skies the UK has to offer. Regular tours and opportunities to view the stars through professional telescopes take place on a nightly basis.
5. **Glasgow Science Centre** - The Centre is home to hundreds of interactive exhibits throughout the three engaging floors

The Midlands and Wales

1. **Electric Mountain** – Snowdonia – Set against a mountainous backdrop is a working pumped storage power station. Take a tour deep into the heart of the mountain and see the turbines spring into action to meet our ever increasing demand for electricity. Take a stroll up on of the UKs highest peaks in the afternoon.
2. **National Space Centre** – Leicester - With six interactive galleries, the UK's largest planetarium, unique 3D Simulator experience, the award-winning National Space Centre in Leicester is an out of this world visitor attraction
3. **Alton Towers** – Staffordshire – Treat yourself to a go on a few rollercoasters whilst discussing Newton's Laws. You may want to download and take these handy rollercoaster physics notes with you http://www.explainthatstuff.com/rollercoasters.html

Southern England

1. **Royal Observatory** – London - Visit the Royal Observatory Greenwich to stand on the historic Prime Meridian of the World, see the home of Greenwich Mean Time (GMT), and explore your place in the universe at London's only planetarium.
2. **Herschel Museum of Astronomy –** Bath – As you walk around the picturesque Roman city – take an hour or two out at the home of one of the great scientists – discoverer of Infra-red radiation and Uranus.
3. **@Bristol –** Bristol - home to the UK's only 3D Planetarium and one of the biggest science centres.
4. **The Royal Institution –** London – The birthplace of many important ideas of modern physics, including Michael Faraday's lectures on electricity. Now home to the RI Christmas lectures and many exhibits of science history.

A Level Physics Transition Baseline Assessment

40 Marks – 40 Minutes

A single piece of graph paper is required for the completion of the assessment.

You may use a calculator.

Question Number	Topic	Score
1	Symbols and Prefixes	/3
2	Standard Form	/4
3	Re-arranging Equations	/3
4	Atomic Structure	/3
5	Recording Data	/3
6	Graphing	/4
7	Forces and Motion	/10
8	Electrical Circuits	/5
9	Waves	/5
		Total /40

Q1 Complete the following table:

Unit prefix	Meaning
k (kilo)	x 1000
	X 0.000001
M (mega)	
N (nano)	

[3]

Q2

a) Write the following numbers into standard form.

i. 0.012

ii. 120000

iii. 0.00000012

[3]

b) Complete the following calculations and right your answers to an appropriate number of significant figures.

i. 2.1 X 0.15

ii. 0.345 ÷ 0.114

[4]

Q3 Re-arrange the following equations to make R the subject of the equation.

a) $\boldsymbol{Q = WERTY}$

b) $\boldsymbol{Q^2 = WR^2}$

c) $\boldsymbol{Q = W - RT^2}$

[3]

Q4 Name the 3 particles (from GCSE) that make up an atom.

…… **[1]**

a) Which one of the above particles is not found in the nucleus of an atom?

…… **[1]**

b) Which of the above particles will be found in varying quantities in the nuclei of isotopes of the same element?

…… **[1]**

Q5

a) Complete the following table

Voltage (__)	___________ (A)		
	Repeat 1	Repeat 2	Average
2	0.23	0.26	0.25
4	0.46	0.53	
6	0.69	0.78	0.74
8	0.92	1.04	0.98
10	1.15	1.30	1.23

[3]

Q6

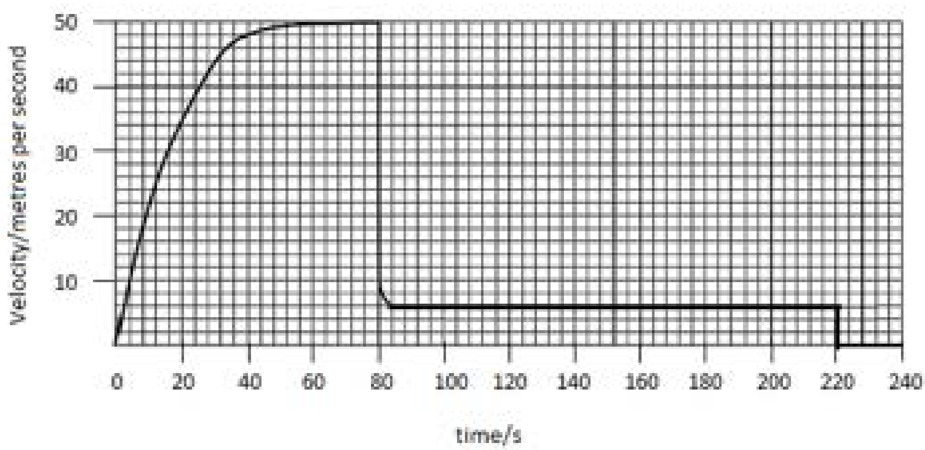
a) Use your piece of graph paper to plot a graph of Current (x-axis) against Voltage (y-axis) drawing a line of best fit through your data points.

[4]

b) Find the gradient of your line of best fit

[3]

Q7 The graph below shows the journey of a skydiver after they have left the plane.



a) Explain the shape of the graph commenting on how and why the forces have changed.

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[6]

b) Calculate the distance travelled whilst at the second terminal velocity.

[2]

c) Calculate the **average** acceleration in the first 20 seconds.

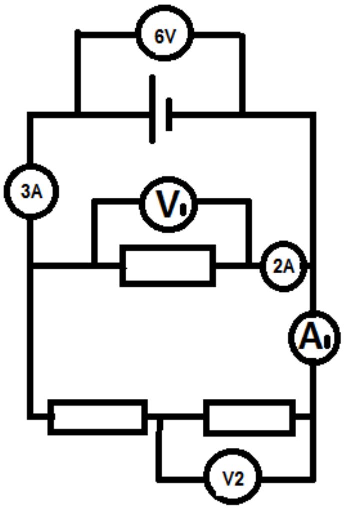
[2]

Q8

a) Draw a circuit diagram to show how the resistance of a filament bulb could be measured using an ammeter and a voltmeter.

[2]

b) Look at the circuit diagram below. All of the resistors are identical.

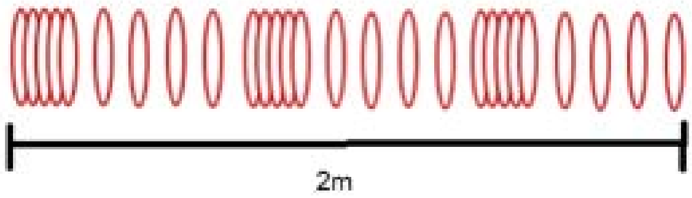


Write the missing values of current and potential difference:

i. V1 =
ii. V2 =
iii. A1 =

[3]

Q9 The diagram below shows a diagram of 3 complete longitudinal wave oscillations on a slinky:



a) State the wavelength of the wave shown

…… **[1]**

b) Label a complete wavelength on the diagram above with the correct symbol used for wavelength in GCSE and A Level Physics

[1]

c) If the above wave had a frequency of 5Hz how long would it take an individual hoop to complete 1 full oscillation?

[1]

d) Calculate the speed of the wave

$$\boldsymbol{wavespeed = frequency \times wavelength}$$

Wave speed = ____________ Unit _______[2] the PiXL club partners in excellence

A Level Physics Baseline Assessment SUGGEST MARKSCHEME

Q1

a)

Unit prefix	Meaning
k (kilo)	x 1000
µ(micro)	X 0.000001
M (mega)	x 1000000
N (nano)	x 0.000000001

[3]

Q2

c) Write the following numbers into standard form

i. 0.012 $\mathbf{1.2 \times 10^{-2}}$

ii. 120000 $\mathbf{1.2 \times 10^{5}}$

iii. 0.00000012 $\mathbf{1.2 \times 10^{-7}}$

[3]

d) Complete the following calculations and right your answers to an appropriate number of significant figures.

i. 2.1 X 0.15
   a. 0.315 = 0.32(2sf)

ii. 0.345 ÷ 0.114
   a. 3.0263…=3.03 (3sf)

Award 1 mark for correct answer and 1 mark for correct number of s.f. [4]

Q3 Re-arrange the following equations to make R the subject of the equation.

a) $\boldsymbol{Q = WERTY}$

$$\boldsymbol{R = \frac{Q}{WETY}}$$

b) $\boldsymbol{Q^2 = WR^2}$

$$\boldsymbol{R = \sqrt{\frac{Q^2}{W}}}$$

c) $\boldsymbol{Q = W - RT^2}$

$$\boldsymbol{R = \frac{W - Q}{T^2}}$$

[3]

Q4

a) Name the 3 particles (from GCSE) that make up an atom

Proton, Neutron, Electron (any order) [1]

b) Which one of the above particles is not found in the nucleus of an atom?

Electron [1]

c) Which of the above particles will be found in varying quantities in the nuclei of isotopes of the same element?

Neutron [1]

Q5

a)

Voltage (V)	Current (A)		
	Repeat 1	Repeat 2	Average
2	0.23	0.26	0.25
4	0.46	0.53	0.50
6	0.69	0.78	0.74
8	0.92	1.04	0.98
10	1.15	1.30	1.23

1 Mark for correct unit (V or volts)

1 Mark for correct heading (Current in Amps or A)

1 Mark for correct average, 1 Mark if rounded to correct number of s.f.

[3]

Q6

a) Use your piece of graph paper to plot a graph of Current (x-axis) against Voltage (y-axis) drawing a line of best fit through your data points.

1 mark if BOTH x and y axis cover half the graph paper

1 mark for correctly labelling x and y axis including units

1 mark if data points are correctly plotted (check 3)

1 mark for correct line of best fit (with even spread of points above and below)

[4]

b) Find the gradient of your line of best fit

Woking must be shown for the award of any marks

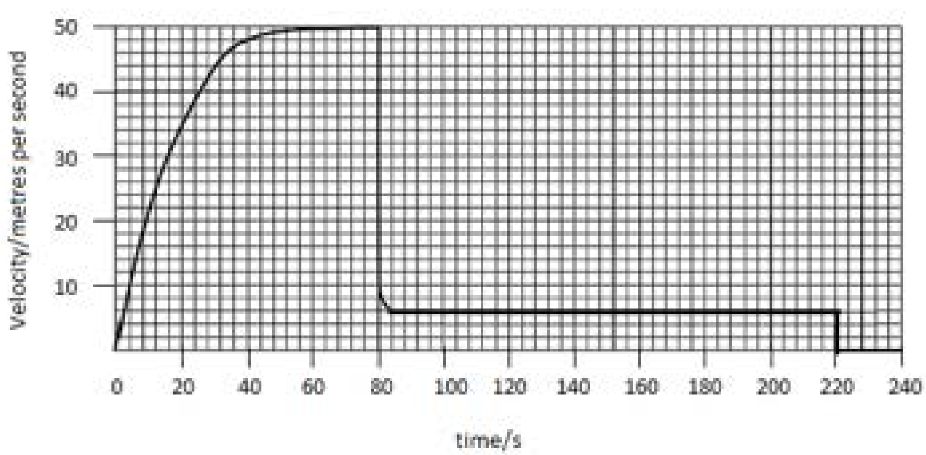
1 mark for correct y axis read offs

1 mark for correct x axis read offs

1 mark for correct calculation of their own gradient

[3]

Q7 The graph below shows the journey of a skydiver after they have left the plane.



a) Explain the shape of the graph commenting on how and why the forces have changed.

Band 1 (1/2 Marks)	Band 2 (3/4 Marks)	Band 3 (5/6 Marks)
Correctly describes the motion of the parachutists. E.g. Acceleration (at reducing rate) Terminal velocity/constant speed, deceleration, lower terminal velocity. There may be small errors in spelling and grammar.	Correctly describes motion and links to the balancing and unbalancing of the forces of weight and drag. Almost faultless spelling and grammar.	Explains why increasing velocity produces increased drag and why opening the parachute produces increase drag, using ideas of collisions of air particles with the surface of the skydiver/parachute. Faultless spelling and grammar

[6]

b) Calculate the distance travelled whilst at the second terminal velocity.

$$(220s - 84s) \times 6ms^{-1} = 816m$$

[2]

c) Calculate the average acceleration in the first 20 seconds.

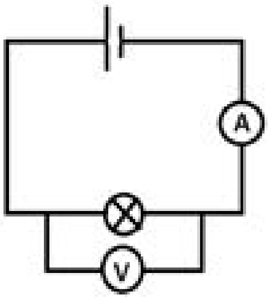
$$\frac{34ms^{-1}}{20s} = 1.7 \text{ [1]} \quad ms^{-2} \text{ [1]}$$

Award 1 mark for correct unit [2]

Q8

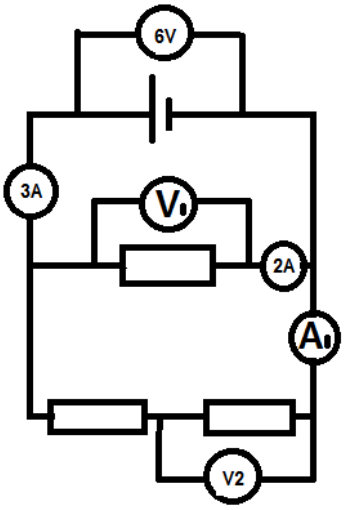
a) Draw a circuit diagram to show how the resistance of a filament bulb could be measured using an ammeter and a voltmeter.

Award 1 mark for correctly positions ammeter [1] and voltmeter [1]



[2]

b) Look at the circuit diagram below. All of the resistors are identical.

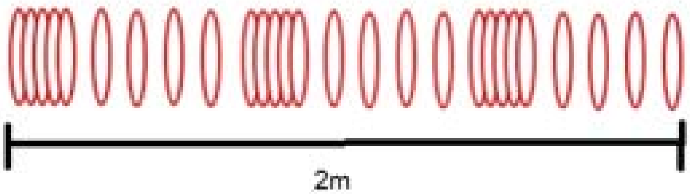


Write the missing values of current and potential difference:

i. V1 = 6V
ii. V2 = 3V
iii. A1 = 1A

[3]

Q9 The diagram below shows a diagram of 3 complete longitudinal wave oscillations on a slinky:



a) State the wavelength of the wave shown

……………………………………………………………2/3m = 0.7m…………………………………………………… [1]

b) Label a complete wavelength on the diagram above with the correct symbol used for wavelength in GCSE and A Level Physics

[1]

c) If the above wave had a frequency of 5Hz how long would it take an individual hoop to complete 1 full oscillation?

0.2s

[1]

d) Calculate the speed of the wave

$$\boldsymbol{wavespeed = frequency \times wavelength}$$

$$\boldsymbol{wavespeed = 5 \times \frac{2}{3} = 3m/s(1sf)}$$

Wave speed = ____________ Unit _______[2]

<u>Preparation for A Level Politics.</u>

<u>A Level Politics:</u>

‘*Man is a political animal*’(Aristotle)

<u>Politics will suit students who:</u>

- have an interest in the world around them: who want to know more about the society they live in, how it works and how it could work,
- enjoy debate, discussion and argument: who are comfortable with the fact that in politics there are no simple ‘rights’ or ‘wrongs’,
- like to think for themselves: who wants to develop their own views, rather than accept the views of others.
- Politics will be a new subject to the majority of you and therefore you may see it as an opportunity to do something fresh and new, an enriching experience in itself.
- It is particularly important that you have an **<u>enquiring mind</u>**,a desire to learn about how the political system works and its impact on you, the citizen.

“Just because you do not take an interest in politics, doesn’t mean politics won’t take an interest in you." –

attributed to Pericles, 5th century BC

<u>The skills you will develop studying politics:</u>

· **Knowledge and understanding** of basic political ideas , concepts, structures and processes, the relationship between them and how they work at different levels.

· An ability to interpret and**analyse**political information in various forms and from various sources, and to applya range of political ideas concepts and theories.
· An ability to **evaluate**arguments, theories, values and ideologies to explain political behaviour and suggest solutions to controversial issues.

· The skills to **organise and present an argument with relevance, clarity and coherence using good English.**

Course Outline:

Year 12:	Year 13
UK Politics: Democracy & participation, political parties, electoral systems, voting behaviour, the media. **Core Politics Ideas:** Liberalism, Conservatism, Socialism. **UK Government:** The Constitution, Parliament, Prime Minister & the Executive, relationships between the branches of government.	**Non-core Political Ideas:** *Feminism* **Government and Politics of the USA:** The US constitution, federalism, US congress, US presidency, US Supreme Court, civil rights, US democracy & participation, comparative theories.
Exam: Paper 1: UK Politics & Core Political Ideas Paper 2: UK Government & Non-Core Political Ideas Paper 3: Comparative Politics (USA)	

How will you be successful on this course?

You will need to take an interest in current affairs and political issues.

The most highly recommended resource of all (because it succeeds so well in providing intelligent analysis which is accessibly written without being at all dumbed down) is https://www.bbc.co.uk/news/politics

Try the other reading/listening/watching recommendations below, with the intention of continuing with what you find useful and enjoyable. Few subjects can offer such entertaining homework.

The following are all quality newspapers which are free to access:

- https://www.theguardian.com/uk
- https://www.independent.co.uk/
- https://www.telegraph.co.uk/
- https://www.nytimes.com/
- http://www.washingtonpost.com/
- https://www.washingtontimes.com/

TV and Radio, which can be accessed, respectively, via

- https://www.bbc.co.uk/iplayer
- http://www.bbc.co.uk/radio
- https://www.itv.com/hub/shows

Podcasts – you can listen to these on multiple apps/websites:

Summer Activities in Preparation for A level Politics:

You will need to download the following booklet:
https://assets-learning.parliament.uk/uploads/2019/12/How-it-Works-booklet.pdf

Read through the booklet and complete the following activities:

Task 1: In your own words write a short definition for each of these keywords/ phrases:

Keyword/ phrase	Definition
Politics	
Suffrage	
Political Participation	
Democracy	
Direct Democracy	
Representative Democracy	
Member of Parliament (MP)	
General Election	
Devolution	
Devolved Assemblies/ Parliament	
By-election	
Referendum	
Manifesto	
First-Past-The-Post (FPTP)	

Constitution	
Uncodified Constitution	
Parliamentary sovereignty	
Legislature	
Executive	
Judiciary	
Role of Parliament	
Parliamentary Reform Acts 1911 and 1949	
Human Rights Act 1998	
Constitutional Reform Act 2005	
Fixed Term Parliament Act 2011	
Bicameral legislature	
House of Commons	
Constituency	
Frontbencher minister	
Backbench MP	
The Speaker	

The Opposition	
House of Lords	
Crossbencher	
Life Peer	
Hereditary Peer	
Select Committee	
Legislative Committee	
Government Bills	
Private Members Bills	
Political Party	
Party Rebel	
Party Whips	
Coalition government	
Partisan	

Task 2: Investigate the role of an MP.

A. research your local MP, who is he/she; which party do they represent; which constituency do they represent; what have they recently shown an interest in; research their voting history to see where they stand on key issues.
https://members.parliament.uk/

B. investigate the role of an MP by playing the game in the link below.
https://learning.parliament.uk/resources/mp-for-a-week/#cta-target

Findings:

a.) My local MP
b.) The role of an MP

Task 3: Watch the news over the summer holidays. Create a 10 question quiz to test the rest of the class when you come back:

Questions:

1.
2.
3.
4.
5.
6.
7.
8.
9.
10.

Task 4:

- Research one pressure group that currently exists in the UK.
- Who are they?
- What type of group are they? (causal, sectional, insider, outsider)
- What are they campaigning for and why?
- What methods have they used to campaign?
- Do you think they have been successful so far?

Example pressure group

Task 5: The age to vote in UK general elections is 18 years of old, so there has been debate in recent years about whether 16 year olds should be allowed to vote in UK general elections.

Read the source below and answer the questions:

Sixteen-year-olds are great fun but they are not grown up. They cannot marry in England and Wales, drive, smoke, get a tattoo or buy alcohol on their own. They are legally classed as children and are supposed to be in formal education or training to 18. Sixteen-year-olds have not taken to the streets demanding the right to the franchise, like the Chartists or suffragettes did.

So why has Keir Starmer suddenly reiterated that he intends to enfranchise them to vote in UK-wide 2024 general election? The only plausible answer is that he hopes they will vote for him. In a desperate attempt to justify this manoeuvre, he says that, if 16-year-old soldiers "are old enough to take a bullet for our country", they should get the vote. (Likewise if they are "old enough to work" and "pay taxes".)

In fact, 16-year-olds are not old enough to "take a bullet", since frontline army service is banned for under-18s. Meanwhile, precious few 16-year-olds work, since Starmer's own party, when last in power, made it supposedly compulsory for them to remain in education or training to 18. As for taxes, younger children already pay VAT when they spend their pocket money.

Opponents also point to the fact that 18- to 24-year-olds have the lowest turnout of any age group in elections, reflecting an apparent lack of interest in politics. In Wales, which lowered the voting age to 16 in 2021 for Senedd and local government elections, early evidence suggests that the new group plans to vote at a similarly low rate to other age groups under 55. Critics question whether an even younger generation would be any different. And there are concerns that teenagers who do want to cast their votes would be impressionable and easily influenced by radical politics, or would not fully think things through and would blindly vote for the same party as their parents.

But calls to lower the voting age come from a range of sources – adults as well as teenagers themselves, backed up by youth organisations, pressure groups and politicians. 16 and 17 year olds in the Isle of Man, Jersey, Guernsey, Brazil and Austria already have the vote. They can also vote in some elections in Germany, Malta and Norway. Evidence from the Scottish independence referendum, substantiated by research from Austria and Norway, shows – aided by the encouragement of families and schools – 16 and 17 year-olds have higher rates of turnout than 18 to 24 year-olds.

For supporters, it's about giving young people a say in matters that directly affect them, such as tuition fees. It's also thought that lowering the limit will encourage civic-mindedness at an earlier age and establish an interest in the political system, which will be continued throughout a person's life. Scotland's positive experience of including 16- and 17-year- olds in the 2014 independence referendum led to the lowering of the voting age for local and Holyrood elections. A study by the University of Edinburgh during the referendum found that some teenagers were initially doubtful of their own abilities to make the right decision, but that this led them to actively seek out information to help inform their judgement. In some cases, teenagers even influenced their parents' voting intentions with their new-found knowledge.

Sources:
https://www.theguardian.com/commentisfree/2025/apr/17/labour-keir-starmer-votes-16-year-olds-sorry-not-convinced?utm_source=chatgpt.com&scrlybrkr=7baa708c
http://www.bbc.co.uk/guides/zsbtbk7
https://www.electoral-reform.org.uk/campaigns/votes-at-16/

1. Using two differently coloured highlighters, indicate:
    a) The arguments in the source that support the view that 16 to 18 year olds should be given the vote
    b) The arguments in the source that support the view that the voting age should remain at 18

2. Match up 3 arguments that support the view that 16 to18 year olds should be given the vote with 3 counter-arguments that support the view that they should not.

Arguments that support the view that 16 to 18 year olds should get the vote.	**Counter-arguments that support the view that the voting age should remain at 18**

You are now going to practice **analysing** one of these arguments. You are going to use **evidence** to back-up your analysis. You may need to do some additional research to answer these question

The work you have just done is how you would start to answer the A Level exam question –

The source states that **the 18-24 age group has the lowest turnout in UK elections so therefore even younger voters are likely to have an even lower turnout.**	
What is turnout?	
For example, what was the turnout for 18-24 year olds in the recent 2024 general election?	
How does this compare with turnout for this age group in the 2019 and 2017 general elections?	
Why therefore is it perhaps not a good idea to extend the vote to 16 and 17 year olds?	
In the 2014 Scottish Independence Referendum (the first UK election to give 16 year olds the vote) what was the turnout for 16-18 years old? Which age group had the lowest turnout in this referendum?	
Therefore, how does this argument prove that the voting age should not be reduced to 16 years old?	

'Using the source, evaluate the view that the voting age should be lowered to 16 for UK general elections.'

Don't worry you are not going to have to write a full answer to this question: you are just going to have a go at writing a conclusion to this question

Answer (what is your decision should 16 years olds get the vote?)	
Substantiate your view. What is the most convincing argument to support your opinion?	

A level Psychology   AQA course codes: 7181/7182

Bridging task Year 12 Psychology

These 2 tasks have been designed for students based on the new AQA GCE A level Psychology specification.

Task 1

'Identification' is a type of conformity. This happens when an individual takes on the views of a group they join or admire. Identification was shown in a famous study by Philip Zimbardo in 1971, which changed the face of social psychology. Working at Stanford University in America, Zimbardo set up a mock prison in the basement of the university over the summer vacation. Zimbardo wished to see if the brutality found in many American prisons at the time was a consequence of the personality of the guards or identification with the social roles in which they were placed.

So serious were the concerns with the findings, the experiment which had been intended to run for two weeks, was called off after six days. The results were interpreted as showing the power of the situation to influence conformity. The implication from these findings was that ordinary, stable individuals can abuse power and behave in violent and anti-social ways if placed in a situation that facilitates this.

Task to be handed in:

Write an essay to discuss how social influence (such as conformity) can have an effect on behaviour. You should use and apply psychological terminology such as compliance, identification and social roles. You must make reference to the Zimbardo experiment but you should also refer to at least one other psychological study or real life event that has occurred in society which have exhibited similar behaviour e.g. Asch's (1951) conformity study or Abu Ghraib prison in Iraq where U.S soldiers tortured inmates.
You should prepare for this by watching “The Stanford Prison Experiment” videos on Youtube, or researching it.

The report must not exceed 500 words. You must work on your own to produce this essay.

Assessment criteria:

Your work will be assessed on your ability to make your point, provide evidence from your research, and explain how the point links back to the question. Your ability to maintain a coherent discussion with information from sources will be assessed, as will your spelling and grammar.

A grade will be given for your work which will be based on a mark out of 100. 40 marks are available for showing knowledge and understanding of the concepts and studies and 60 marks for applying them to your discussion of the ways in which social influence can have an effect on behaviour.

Task 2

Pick one of the following Psychological disorders:

- Depression
- OCD
- Phobias

Write a case study of a patient suffering from one of these conditions. It must be 500 words long, and be split into the following sections:

a) Symptoms that the patient is suffering (40 marks)
b) A treatment you will suggest to the patient (10 marks)
c) Research on the effectiveness of this treatment, e.g. what studies have been carried out into this treatment? Does it work? Are there side effects? Etc. (50 marks)

This task will also be graded out of 100.

Context	The following task aims to get you thinking and writing about the importance of society and the influence it has on behaviour. Feral Children are children who have not been brought up by humans so have not been socialised in the way we would expect. They raise important questions... 1. Why do humans act the way we do? 2. How would we act if we were not raised by other humans? 3. Is our behaviour genetic or learnt? The process through which we learn about society is called socialisation and this can influence many aspects of how we act; our expected behaviour as a child, as a male or female and even our expectations of falling in love. You need to be able to understand the process of socialisation and how it influences us.
Wider readings / viewings	The following online articles will give you a glimpse at some of the basic ideas on the issues of socialisation and feral children. The last one on Feral Children contains a 14minute video at the end that you should watch. Socialisation http://www.open.edu/openlearn/body-mind/childhood-youth/childhood-and-youthstudies/childhood/what-socialisation http://anthro.palomar.edu/social/soc_1.htm Feral Children http://listverse.com/2008/03/07/10-modern-cases-of-feral-children/ https://www.telegraph.co.uk/culture/tvandradio/3653890/Cry-of-an-enfantsauvage.html https://www.youtube.com/watch?v=nv3ocntSSUU Gender Socialisation https://www.sparknotes.com/sociology/socialization/section4.rhtml http://www.unicef.org/earlychildhood/index_40749.html
Written task 1	You are to write two 500 word discussions on the following: 1. How do Feral Children provide evidence that human behaviour is learnt rather than genetic? Use Oxana Malaya's behaviour to provide examples to support your answer. 2. 'Gender is learnt'. Provide examples of gender role socialisation that suggest men and women learn how to act in a 'masculine' and 'feminine' way. Do you agree or disagree with the statement?

There are a lot of key terms that you need to know for your exams and to make sure you write great essays! Lets start off with the basics:

<u>Task 2</u>: define the following sociological key terms:

1. Norms
2. Values
3. Socialisation
4. Society
5. Culture
6. Identity
7. Social differentiation
8. Stratification
9. Status
10. Subculture
11. Cultural diversity
12. Consensus
13. Conflict

Now for each of the key terms add a picture and an example to go with them

<u>Task 3:</u>

As a sociology student you will also be expected to keep up to date with the news and current events – research a social issue that interests you!

1. Explain what why that particular issue (what interests you about it?)
2. Create a media collage of articles on your issue and annotate them
3. Apply what the theories might say about the issue!
4. Any research or policy laws which deals with your issue? Name and explain them!

<u>Task 4: How do</u> *you* <u>see society?</u>

Draw (or build a model) of how you view society (use shapes / key words / images where possible)

Provide a written explanation of your work (**<u>At least</u>** a side of A4) to explain your work. What are main issues/ problems/ solutions you feel as a society we face and why!

<u>Task 5: watch some TV!</u>

Sociology is everywhere, so you'll be able to see aspects of it in all forms of media. Try searching for the following titles and make notes on any videos you watch, try to keep it relevant to the topics taught in Sociology (see above).

- School Swap documentary
- Stacey Dooley documentaries

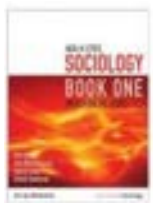
(BBC iPlayer/YouTube)

- Black Mirror (Netflix)
- Reggie Yates documentaries
- Dispatches documentaries (4OD)
- Panorama documentaries (BBC iPlayer)

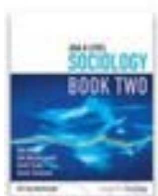
Useful Resources & Further Reading

To help you on your way…

It is also recommended that you take an active interest in the news and what is going on around you in the wider world as a lot of the concepts and ideas covered in lessons will relate to what is going on. The ability to be able to draw upon contemporary examples will also help illustrate your application and understanding skills which will be vital in order for you to achieve the higher grades in this subject.



The main textbook used in AQA A level Sociology AS and Year 1. Written by Robb Webb et al
Napier press

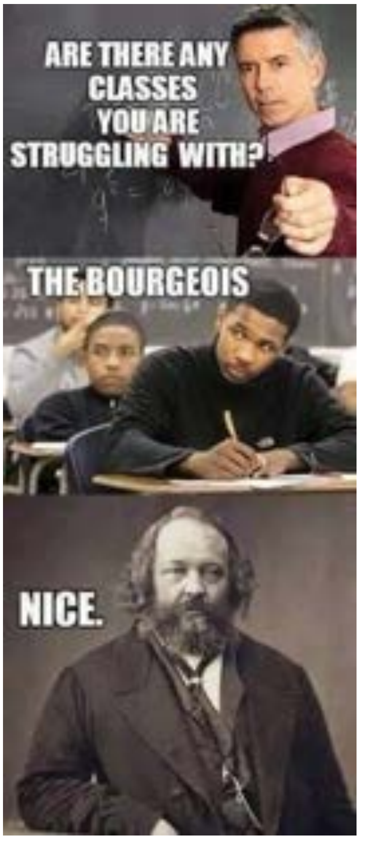


The main textbook used in AQA A level Year 2. Written by Robb Webb et al
Napier press

Websites:

- AQA New Specification – http://www.aqa.org.uk/subjects/sociology/as-and-alevel/sociology-7191-7192/introduction
- AQA - Example Assessment Material -http://www.aqa.org.uk/subjects/sociology/as-anda-level/sociology-7191-7192/assessment-resources
- S-Cool Revision Materials - http://www.s-cool.co.uk/a-level/sociology
- Every Day Sexism Project -http://everydaysexism.com/index.php/about
- Who Needs Feminism - http://whoneedsfeminism.com/about.html
- He for She Campaign - http://www.heforshe.org/

- https://www.tutor2u.net/sociology/reference
  http://politybooks.com/kenbrowne/resources.asp
- https://napierpress.com/book-one-workbooks
- https://napierpress.com/book-two-workbooks
- http://www.earlhamsociologypages.co.uk
- https://www.sociologystuff.com



For your first lesson… We expect you to have the following:

- A level arched folder – labelled with your name on and Sociology (spelt correctly).
- There must be dividers in your folder – plastic ones are better as they are tougher.
- Your own lined paper • Plenty of plastic wallets. • A pencil case with the usual but we use a lot of post-it notes and highlighters so have plenty of those.
- A copy of the Year 1 textbook
- Your task homework contained in this booklet.

Well done! You're all set ready to embark on your Sociology A Level course. We look forward to seeing you in September!